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4. Find the directional derivative of $f(x,y,z) = xy+z^2$ at $(1,1,1)$ in the direction toward $(5,-3,3)$.

$\vec{\nabla} f = y\vec{i} + x\vec{j} + 2z\vec{k}$

$\vec{\nabla} f|_{(1,1,1)} = \vec{i} + \vec{j} + 2\vec{k}$

$\vec{u} = \dfrac{4\vec{i} - 4\vec{j} + 2\vec{k}}{\sqrt{16+16+4}}$

$D_{\vec{u}} f = \vec{\nabla} f|_{(1,1,1)} \cdot \vec{u} = (\vec{i}+\vec{j}+2\vec{k}) \cdot \dfrac{(4\vec{i}-4\vec{j}+2\vec{k})}{6} = \dfrac{4-4+4}{6} = \dfrac{2}{3}$

5. Let $f(x,y) = \dfrac{xy^2}{3x^2+2y^4}$.

(a) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along every straight line of the form $y = mx$.

$\lim\limits_{\substack{(x,y)\to(0,0)\\ \text{along } y=mx}} f = \lim\limits_{x\to 0} \dfrac{x(mx)^2}{3x^2+2(mx)^4} = \lim\limits_{x\to 0} \dfrac{mx^3}{3x^2+2m^4x^4} = \lim\limits_{x\to 0} \dfrac{mx}{3+2m^4x^2} = 0$

(b) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the parabola $x = y^2$.

$\lim\limits_{\substack{(x,y)\to(0,0)\\ \text{along } x=y^2}} f = \lim\limits_{x\to 0} \dfrac{y^2y^2}{3y^4+2y^4} = \lim\limits_{x\to 0} \dfrac{y^4}{5y^4} = \lim\limits_{x\to 0} \dfrac{1}{5} = \dfrac{1}{5}$

(c) What is $\lim\limits_{(x,y)\to(0,0)} f(x,y)$?

This limit does not exist.