Math 241 Spring 1998 Exam 3

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Let $f(x,y,z) = xz\ln(x+y+z)$. Find $\vec{\nabla} f$.

$$\vec{\nabla} f = \left(\frac{xz}{x+y+z} + z\ln(x+y+z)\right)\vec{i} + \frac{xz}{x+y+z}\vec{j} + \left(\frac{xz}{x+y+z} + x\ln(x+y+z)\right)\vec{k}$$

2. Find the equation of the plane tangent to $z = x^2 + y^2$ at $(1,1,2)$.

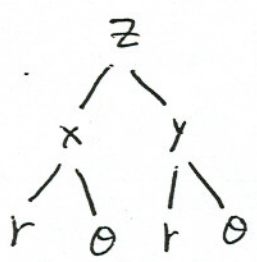
the surface is the level set $0 = x^2+y^2-z$

$\vec{\nabla}(RHS) = 2x\vec{i} + 2y\vec{j} - \vec{k}$

$\vec{\nabla}(RHS)\big|_{(1,1,2)} = 2\vec{i} + 2\vec{j} - \vec{k}$

$$2(x-1) + 2(y-1) - (z-2) = 0$$

3. Suppose that $z = f(x,y)$, and x and y are writen polar coordinates (that is, $x = r\cos\theta$ and $y = r\sin\theta$). Express $\dfrac{\partial z}{\partial \theta}$ in terms of $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.



$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial \theta}$$

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x}(-r\sin\theta) + \frac{\partial z}{\partial y}r\cos\theta$$