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8. The temperature of a plate at the point (x, y) is $T(x, y) = 100 + x^2 - y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(5, \sqrt{75})$. (The particle always moves in the direction of the greatest increase in temperature.)

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ be the path of the particle

We know that the particle moves in the direction of $\vec{\nabla} T$, so

$\vec{r}'(t) = c(t)\,\vec{\nabla} T$ for some scalar function $c(t)$

$x'(t)\vec{i} + y'(t)\vec{j} = c(t) \cdot (2x\vec{i} - 2y\vec{j})$

So $x' = 2cx$, $y' = -2cy$ where x, x', y, y', and c all are functions of t.

$\frac{x'}{x} = 2c$

$-\frac{y'}{y} = 2c$

So $\frac{x'}{x} = -\frac{y'}{y}$

Integrate each side with respect to t

$\int \frac{x'(t)}{x(t)}\,dt = -\int \frac{y'(t)}{y(t)}\,dt$

So $\ln|x(t)| = -\ln|y(t)| + K$

Exponentiate each side

$e^{\ln|x|} = e^{-\ln|y| + K}$

(as before x and y are functions of t.)

$|x| = \frac{1}{|y|} e^K$

$x = \frac{\pm e^K}{y}$

$y = \frac{\pm e^K}{x}$ Let $k = \pm e^K$

$y = \frac{k}{x}$ when $x = 5$ $y = \sqrt{75}$

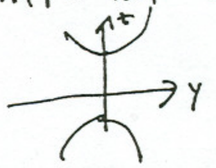
so $\sqrt{75} = \frac{k}{5}$

$5\sqrt{75} = k$

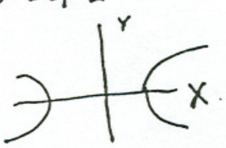
$y = \frac{5\sqrt{75}}{x}$

9. Sketch and name $x^2 - y^2 + z^2 = 1$ in 3-space.

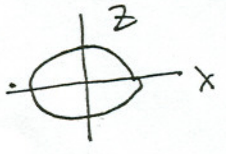
when $x = 0$ the graph is a hyperbola $z^2 - y^2 = 1$

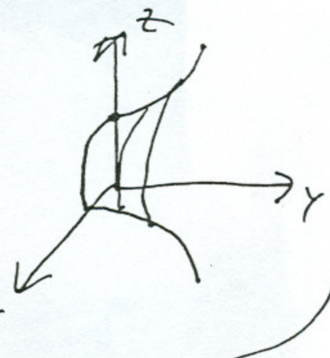


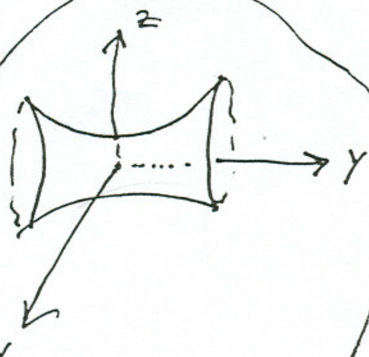
when $z = 0$ the graph is a hyperbola



when $y = 0$ the graph is a circle







The graph is a hyperboloid of one sheet