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9. Find the equations of the line tangent to the curve parameterized by $\vec{r}(t) = t^2\vec{i} + 3t\vec{j} + 6t^{2}\vec{k}$ at the point $(4,6,24)$.

The curve hits the point at $t=2$.

$\vec{r}'(t) = 2t\vec{i} + 3\vec{j} + 12t\vec{k}$

$\vec{r}'(2) = 4\vec{i} + 3\vec{j} + 24\vec{k}$

(The coefficient of $\vec{k}$ is $6t^2$.)

tan line is

$x = 4 + 4\tau$
$y = 6 + 3\tau$
$z = 24 + 24\tau$

10. The temperature of a plate at the point (x, y) is $T(x,y) = 20 - 2x^2 - y^2$. Find the path that a heat seeking particle would travel if it starts at the point $(1,2)$. (The particle always moves in the direction of the greatest increase in temperature.)

Let $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$ parameterize the path of the particle. We are told that $\vec{r}'(t) = c(t)\,\vec{\nabla} T|_{(x(t),y(t))}$ for all t

so $x'(t)\vec{i} + y'(t)\vec{j} = c(t)(-4x\vec{i} - 2y\vec{j})|_{(x(t),y(t))}$

$x'(t)\vec{i} + y'(t)\vec{j} = c(t)(-4x(t)\vec{i} - 2y(t)\vec{j})$

$\begin{cases} x' = -4cx \\ y' = -2cy \end{cases}$

$\therefore \frac{x'}{-4x} = c = \frac{y'}{-2y}$

so $\frac{x'}{-4x} = \frac{y'}{-2y}$

so $-\frac{1}{4}\int \frac{x'(t)}{x(t)}\,dt = -\frac{1}{2}\int \frac{y'(t)}{y(t)}\,dt$

Multiply both sides by -4. Integrate.

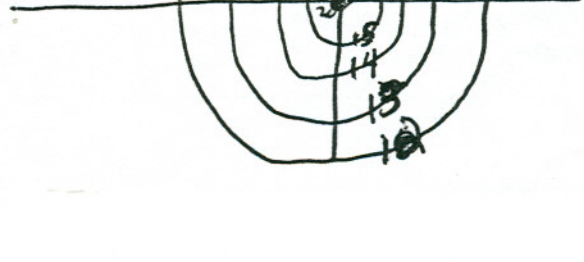
$\ln|x(t)| = 2\ln|y(t)| + C$

$x = \pm e^C y^2$

so $x = Ky^2$ for some constant K

when $x=1$, $y=2$ so $1 = K4$ and $K = \frac{1}{4}$

so $x = \frac{1}{4}y^2$



The level set are ellipses. The particle heads toward the origin.

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