

5. A boy's toy boat slips from his grasp at the edge of a straight river. The stream carries it along at 5 feet per second. A crosswind blows it toward the opposite shore at 4 feet per second. If the boy runs along the shore at 3 feet per second following his boat, how fast is the boat moving away from him when $t = 3$ seconds.



$$\text{so } D = \sqrt{20t^2}$$

$$D = \sqrt{20}t$$

The question asks for $\left. \frac{dD}{dt} \right|_{t=3}$

and this is $\sqrt{20} \text{ ft/s}$

The position of the Boy at time t is $(0, 3t)$

The position of the boat at time t is $(4t, 5t)$

The distance from the boy to the boat at time t is $D = \sqrt{(4t)^2 + (5t - 3t)^2}$

6. Find all local maximum points, all local minimum points, and all saddle points of $f(x, y) = 2x^4 - x^2 + 3y^2$.

$$f_x = 8x^3 - 2x = 2x(4x^2 - 1) = 2x(2x-1)(2x+1)$$

$$f_y = 6y$$

so $f_x = 0$ and $f_y = 0$ at $(0,0)$, $(\frac{1}{2}, 0)$, $(-\frac{1}{2}, 0)$

$$f_{xx} = 24x^2 - 2$$

$$f_{xy} = 0$$

$$f_{yy} = 6$$

2nd D test at $(0,0)$

$$f_{xx}f_{yy} - (f_{xy})^2$$

$$D|_{(0,0)} = (-2)(6) < 0$$

so $(0,0,0)$ is a saddle pt

2nd D test at $(\frac{1}{2}, 0)$

$$D|_{(\frac{1}{2}, 0)} = \left(\frac{24}{4} - 2\right)(6) > 0$$

$$f_{xx}|_{(\frac{1}{2}, 0)} > 0$$

so $(\frac{1}{2}, 0, f(\frac{1}{2}, 0))$ is a min

2nd D test at $(-\frac{1}{2}, 0)$

$$D|_{(-\frac{1}{2}, 0)} = 4(6) > 0$$

$$f_{xx}|_{(-\frac{1}{2}, 0)} > 0$$

$(-\frac{1}{2}, 0, f(-\frac{1}{2}, 0))$ is a local max