11 69

2. Let $f(x,y) = x\ln(x^2+y^2)$. Find $\vec{\nabla} f(1,2)$.

$$f = \left[x\frac{2x}{x^2+y^2} + \ln(x^2+y^2)\right]\vec{i} + \frac{2xy}{x^2+y^2}\vec{j}$$

$$\vec{\nabla} f(1,2) = \left(\frac{2}{5} + \ln 5\right)\vec{i} + \frac{4}{5}\vec{j}$$

3. Find the directional derivative of $f(x,y) = y^2 \ln x$ at the point $(1,2)$ in the direction of $\vec{a} = \vec{i} - \vec{j}$. so $\vec{u} = \frac{\vec{i}-\vec{j}}{\sqrt{2}}$

$$D_{\vec{u}} f = \vec{\nabla} f\Big|_{(1,2)} \cdot \vec{u} = \left(\frac{y^2}{x}\vec{i} + 2y\ln x\,\vec{j}\right)\Big|_{(1,2)} \cdot \vec{u} = 4\vec{i} \cdot \left(\frac{\vec{i}-\vec{j}}{\sqrt{2}}\right)$$

$$= \frac{4}{\sqrt{2}}$$

4. Find the equation of the plane tangent to the surface $z = x^3y + 3xy^2$ at the point where $x = 2$ and $y = -2$.

Gradients are $\perp$ level sets.

view the surface as $0 = x^3y + 3xy^2 - z$

$$\vec{\nabla} RHS = (3x^2y + 3y^2)\vec{i} + (x^3 + 6xy)\vec{j} - \vec{k}$$

$$\vec{\nabla} RHS\Big|_{(2,-2)} = (-3(4)(2) + 12)\vec{i} + (8 - 24)\vec{j} - \vec{k}$$

$$= -12\vec{i} - 16\vec{j} - \vec{k}$$

The z-coordinate of my point is

$$z = -16 + 3(8) = 8$$

$$0 = -12(x-2) - 16(y+2) - (z-$$