71 67 44

8. Let $f(x,y) = \dfrac{xy^2}{2x^2+y^4}$.

(a) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along every straight line of the form $y = mx$.

$\lim_{(x,y)\to(0,0) \text{ along } y=mx} f = \lim_{x\to 0} \dfrac{x(mx)^2}{2x^2+(mx)^4} = \lim_{x\to 0} \dfrac{m^2x^3}{2x^2+m^4x^4} = \lim_{x\to 0} \dfrac{m^2x}{2+m^2x^4}$

$= 0$

(b) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along the parabola $x = y^2$.

$\lim_{(x,y)\to(0,0) \text{ along } x=y^2} f = \lim_{y\to 0} \dfrac{y^4}{2y^4+y^4} = \lim_{y\to 0} \dfrac{1}{3} = \dfrac{1}{3}$

(c) What is $\lim_{(x,y)\to(0,0)} f(x,y)$? No limit.