Math 241, Fall 1997, exam 3

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PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. CIRCLE your answer. **NO CALCULATORS!** CHECK your answer, whenever possible.

1. Let $f(x,y) = e^{xy}\sin x + 2xy^2$. Find $\vec{\nabla} f$.

$\vec{\nabla} f = (e^{xy}\cos x + y e^{xy}\sin x + 2y^2)\vec{i} + (x e^{xy}\sin x + 4xy)\vec{j}$

2. Find the equation of the plane tangent to $z^2 = 3x^2 + 6y^2$ at $(1,-1,3)$.

$0 = 3x^2 + 6y^2 - z^2$

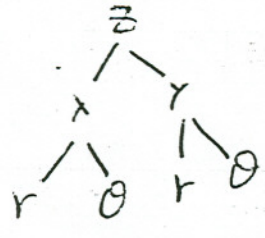
$\vec{\nabla}$ ⊥ level sets!

$\vec{\nabla} = 6x\vec{i} + 12y\vec{j} - 2z\vec{k}$

$\vec{\nabla}|_{(1,-1,3)} = 6\vec{i} - 12\vec{j} - 6\vec{k}$

$6(x-1) - 12(y+1) - 6(z-3) = 0$

3. Suppose that $z = f(x,y)$, and x and y are writen polar coordinates (that is, $x = r\cos\theta$ and $y = r\sin\theta$). Express $\frac{\partial z}{\partial r}$ in terms of $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.



$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial r}$

$\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta$