97

(1,2,3)

9. Find the point on $3x - 5y + 2z = 37$ which is closest to $(1, 2, 3)$.

The line through (1,2,3) + to the plane is

$$\begin{cases} X = 1+3t \\ Y = 2-5t \\ Z = 3+2t \end{cases}$$

This line hits the plane when

$3(1+3t) - 5(2-5t) + 2(3+2t) = 37$

$9t + 25t + 4t + 3 - 10 + 6 = 37$

$38t = 37 + 1$

$38t = 38$

$t = 1$

The line hits the plane at

$(4, -3, 5)$

✓! $(4, -3, 5)$ is on the plane because

$12 + 15 + 10 = 37$ ✓

the distance from $(4, -3, 5)$ to $(1, 2, 3)$ is

$\sqrt{9 + 25 + 4} = \sqrt{38}$ ←

The distance from $(1, 2, 3)$ to the plane is

$$\frac{|3 - 10 + 6 - 37|}{\sqrt{9 + 25 + 4}} = \frac{|-38|}{\sqrt{38}} = \sqrt{38}$$ ← ege

10. Find the point of intersection of the following lines. **CHECK YOUR ANSWER!**

$$\frac{x-5}{1} = \frac{y-5}{3} = \frac{z-7}{4} \quad \text{and} \quad \frac{x+3}{2} = \frac{y+10}{3} = \frac{z+4}{1}.$$

walk on the line on the left

at time t $X = t + 5$, $Y = 3t + 5$, $Z = 4t + 7$

You hit line t when

$$\frac{(t+5)+3}{2} = \frac{(3t+5)+10}{3} = \frac{(4t+7)+4}{1}$$

i.e. $\begin{cases} \frac{t+8}{2} = \frac{3t+15}{3} \\ \frac{t+8}{2} = \frac{4t+11}{1} \end{cases}$

So $\begin{cases} 3t + 24 = 6t + 30 \\ t + 8 = 8t + 22 \end{cases}$

So $\begin{cases} -6 = 3t \\ -14 = 7t \end{cases}$

So $\begin{cases} -2 = t \\ -2 = t \end{cases}$

at $t = -2$ You stand on

$(3, -1, -1)$

Check

$(3, -1, -1)$ is on the 1st line because

$\frac{-2}{1} = \frac{-6}{3} = \frac{-8}{4}$ ✓

$(3, -1, -1)$ is on the second line because

$\frac{6}{2} = \frac{9}{3} = \frac{3}{1}$ ✓