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7. Suppose $\overrightarrow{r}''(t) = \overrightarrow{i} + e^{-t}\overrightarrow{j}$, $\overrightarrow{r}'(0) = 2\overrightarrow{i} + \overrightarrow{j}$, and $\overrightarrow{r}(0) = \overrightarrow{i} + \overrightarrow{j}$. Find $\overrightarrow{r}(t)$.

$\vec{r}'(t) = t\vec{i} - e^{-t}\vec{j} + \vec{c}_1$

$2\vec{i} + \vec{j} = \vec{r}'(0) = -\vec{j} + \vec{c}_1$

$2\vec{i} + 2\vec{j} = \vec{c}_1$

$\vec{r}'(t) = (t+2)\vec{i} + (2 - e^{-t})\vec{j}$

$\vec{r}(t) = \left(\frac{t^2}{2} + 2t\right)\vec{i} + (2t + e^{-t})\vec{j} + \vec{c}_2$

$\vec{i} + \vec{j} = \vec{r}(0) = \vec{j} + \vec{c}_2$

$\vec{i} = \vec{c}_2$

$\vec{r}(t) = \left(\frac{t^2}{2} + 2t + 1\right)\vec{i} + (2t + e^{-t})\vec{j}$

✓

$\vec{r}(0) = \vec{i} + \vec{j}$ ✓

$\vec{r}'(t) = (t+2)\vec{i} + (2 - e^{-t})\vec{j}$

$\vec{r}'(0) = 2\vec{i} + \vec{j}$ ✓

$\vec{r}''(t) = \vec{i} + e^{-t}\vec{j}$ ✓

8. Consider the curve $\overrightarrow{r}(t) = 2\sin t\,\overrightarrow{i} + 3\cos t\,\overrightarrow{j}$.
   (a) Eliminate the parameter and find an equation for this curve which involves only x and y .
   (b) Sketch the curve.
   (c) Which point on the curve corresponds to $t = -\frac{\pi}{4}$.
   (d) Graph $\overrightarrow{r}'(-\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).
   (e) Graph $\overrightarrow{r}''(-\frac{\pi}{4})$. Put the tail of your vector on your answer to (c).

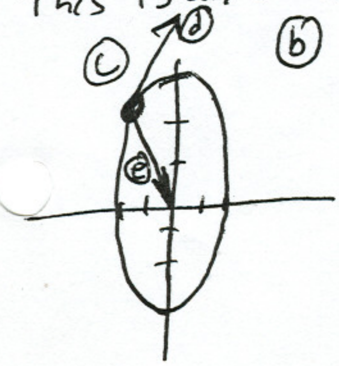
$\begin{cases} X = 2\sin t \\ Y = 3\cos t \end{cases}$

$\sin^2 t + \cos^2 t = 1$

becomes

$\frac{X^2}{4} + \frac{Y^2}{9} = 1$ (a)

This is an ellipse



(c) at $t = -\frac{\pi}{4}$ $\quad X = 2\sin\left(-\frac{\pi}{4}\right) = 2\frac{\sqrt{2}}{2} = -\sqrt{2}$

$Y = 3\cos\left(\frac{-\pi}{4}\right) = 3\frac{\sqrt{2}}{2} = 3\frac{\sqrt{2}}{2}$

(d) $\vec{r}' = 2\cos t\,\vec{j} + 3\sin t\,\vec{j}$

$\vec{r}'\left(-\frac{\pi}{4}\right) = \sqrt{2}\,\vec{i} - \frac{3\sqrt{2}}{2}\vec{j}$

(e) $\vec{r}'' = -2\sin t\,\vec{i} - 3\cos t\,\vec{j}$

$\vec{r}''\left(-\frac{\pi}{4}\right) = \sqrt{2}\,\vec{i} - \frac{3\sqrt{2}}{2}\vec{j}$