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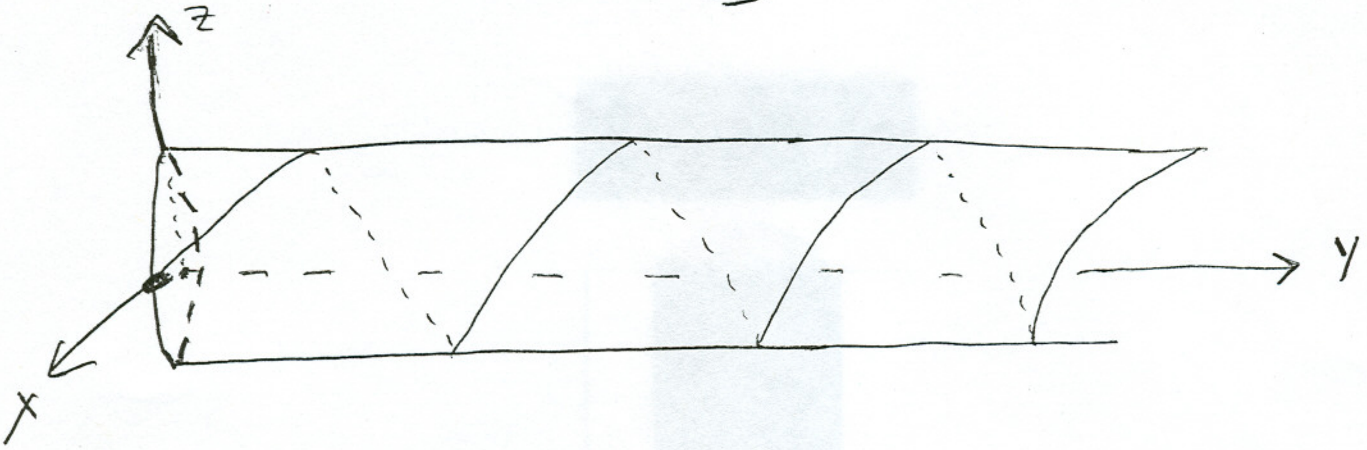
3. Graph and describe the graph of the curve $\vec{r}(t) = \cos t\,\vec{i} + t\,\vec{j} + \sin t\,\vec{k}$ in 3− space.

this is a helix around the Y-axis. If you only think about X and z the graph looks like a circle but y is growing



4. Find the length of the curve $\vec{r}(t) = t^2\,\vec{i} - 2t^3\,\vec{j} + 6t^3\,\vec{k}$ for $0 \le t \le 1$.

Length $= \int_0^1 \|\vec{r}\,'(t)\|\,dt = \int_0^1 \|2t\vec{i} - 6t\vec{j} + 18t^2\vec{k}\|\,dt$

$= \int_0^1 \sqrt{4t^2 + 36t^4 + (18)^2 t^4}\,dt = \int_0^1 2t\sqrt{1 + 9t^2 + 81t^2}\,dt$

$(18)^2 = 4(9)^2$

I pulled $4t^2$ out from each term

$= \int_0^1 2t\sqrt{1+90t^2}\,dt = \frac{1}{90}\int_{t=0}^{t=1} u^{\frac{1}{2}}\,du = \frac{1}{90}\left(1+90t^2\right)^{\frac{3}{2}}\frac{2}{3}\Big]_0^1$

$u = 1+90t^2$

$du = 180t\,dt$

$= \frac{2}{90\,3}\left((91)^{\frac{3}{2}} - 1\right)$