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7. Find the slope of the line tangent to the curve which is the intersection of the surface $36z = 4x^2 + 9y^2$ and the plane $x = 3$ at the point $(3, 2, 2)$.

We want $\left.\frac{\partial z}{\partial y}\right|_{(3,2,2)} = \left.\frac{1}{36}\,18y\right|_{(3,2,2)} = \frac{18}{36}(2) = (1)$

8. A fly is crawling along a wire helix so that its position vector is

$$\vec{r} = 6\cos \pi t\, \vec{i} + 6\sin \pi t\, \vec{j} + 2t\, \vec{k}$$

for $0 \le t$. At what point will the fly hit the sphere $x^2 + y^2 + z^2 = 100$ and how far did it travel in getting there (assuming it started when $t = 0$)?

The fly hits the sphere when

$(6\cos \pi t)^2 + (6\sin \pi t)^2 + (2t)^2 = 100$

$36\cos^2 \pi t + 36\sin^2 \pi t + 4t^2 = 100$

$36 + 4t^2 = 100$

$4t^2 = 64$

$t^2 = 16$

$t = +4$ since $t \ge 0$.

The fly hits the sphere at

$(6\cos 4\pi, 6\sin 4\pi, 8) = (6, 0, 8)$

distance traveled $= \int_0^4 \|\vec{r}'(t)\|\, dt =$

$\int_0^4 \| -6\pi \sin t\, \vec{i} + 6\pi \cos t\, \vec{j} + 2\vec{k} \|\, dt$

$= \int_0^4 \sqrt{36\pi^2 \sin^2 \pi t + 36\pi^2 \cos^2 \pi t + 4}\, dt$

$= \int_0^4 \sqrt{36\pi^2 + 4}\, dt$

$= (4\sqrt{36\pi^2 + 4})$