85

5. (15 points) Let $f(x,y) = \dfrac{xy+y^4}{x^2+y^2}$.

   (a) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along $x = 0$.

   (b) Calculate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along $y = x$.

   (c) What is $\lim_{(x,y)\to(0,0)} f(x,y)$?

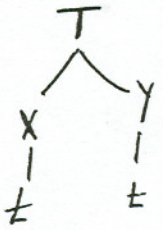
(a) $\lim_{y\to 0} \frac{y^4}{y^2} = \lim_{y\to 0} y^2 = 0$

(b) $\lim_{x\to 0} \frac{x^2+x^4}{x^2+x^2} = \lim_{x\to 0} \frac{x^2(1+x^2)}{2x^2} = \lim_{x\to 0} \frac{1+x^2}{2} = \frac{1}{2}$

(c) This limit Does not exist because (a) and (b) have different answers.

6. The temperature of a metal plate at (x,y) is e^{-x-3y} degrees. A bug is walking northeast at a rate of $\sqrt{8}$ feet per minute (that is, $\frac{dx}{dt} = 2$ and $\frac{dy}{dt} = 2$). From the bug's point of view, how is the temperature changing with time as it crosses the origin?

Let $T(x,y) = e^{-x-3y}$. We want $\left.\dfrac{dT}{dt}\right|_{\substack{x=0\\y=0}}$.



$\frac{dT}{dt} = \frac{\partial T}{\partial x}\frac{dx}{dt} + \frac{\partial T}{\partial y}\frac{dy}{dt} = -e^{-x-3y}\,2 - 3e^{-x-3y}\,2$

$\left.\dfrac{dT}{dt}\right|_{\substack{x=0\\y=0}} = -2-6 = -8$ degrees/min