Math 241, Exam 3, Fall, 2024

YOU SHOULD KEEP THIS PIECE OF PAPER. Write everything on the **blank paper provided**. Return the problems **IN ORDER** (use as much paper as necessary), use **ONLY ONE SIDE** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. (I will return your exam in the next class.) **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) Consider the function $f(x, y) = y - x^2$ and the point $P = (2, 2)$.

   (a) Find the gradient of f at P.

   (b) Find the directional derivative of f in the direction of $\overrightarrow{\boldsymbol{v}} = \overrightarrow{\boldsymbol{i}} + 2\overrightarrow{\boldsymbol{j}}$ at P.

   (c) Draw the level set of f that contains P.

   (d) Draw the gradient of f at P; put the tail of the gradient on P.

(2) Find the length of the curve $\overrightarrow{\boldsymbol{r}}(t) = \cos t\,\overrightarrow{\boldsymbol{i}} + \sin t\,\overrightarrow{\boldsymbol{j}} + t\,\overrightarrow{\boldsymbol{k}}$, for $0 \leq t \leq \pi$.

(3) Find all local maximum points, local minimum points, and saddle points of $f(x, y) = x^2y + 4xy - 2y^2$.

(4) Find the absolute maximum and absolute minimum values of the function

$$f(x, y) = -x^2 - y^2 + 2x + 2y + 1$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 2 - x$.

(5) Find the equation of the plane tangent to $z = x^2 + y^2$ at $(x, y) = (1, 2)$.