Math 241, Exam 3, Fall, 2017 1:15 class Solutions

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Tuesday.

No Calculators, Cell phones, computers, notes, etc.

(1) **Graph, name, describe the set of points in 3-space which satisfy**

$$z^2 - x^2 - y^2 = 1.$$

See the picture page.

(2) **The position vector of an object at time t is**

$$\vec{\boldsymbol{r}}(t) = (3\cos t - 1)\vec{\boldsymbol{i}} + (4\sin t - 2)\vec{\boldsymbol{j}}.$$

Eliminate the parameter and give the path of the object.

We are told that $x = 3\cos t - 1$ and $y = 4\sin t - 2$. We use $\sin^2 t + \cos^2 t = 1$ to see that

$$\boxed{\frac{(y+2)^2}{16} + \frac{(x+1)^2}{9} = 1.}$$

This is an ellipse. There is a picture on the picture page.

(3) **Find the equation of the plane tangent to $z = x^2 + y^2$ when $x = 3$ and $y = 1$.**

Gradients are perpendicular to level sets. We view the paraboloid as the level set $0 = -z + x^2 + y^2$. The gradient of the right side evaluated at the point $(3, 1, 10)$ is

$$(2x\vec{\boldsymbol{i}} + 2y\vec{\boldsymbol{j}} - \vec{\boldsymbol{k}})|_{(3,1,10)} = 6\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} - \vec{\boldsymbol{k}}.$$

The tangent plane is

$$\boxed{6(x-3) + 2(y-1) - (z-10) = 0}.$$

(4) **Find the maximum and minimum values of the function** $f(x,y) = 3x + 4y$ **on the circle** $x^2 + y^2 = 1$**.**

We use the method of Lagrange multipliers and find all points on $x^2 + y^2 = 1$ where the gradient of f is λ times the gradient of $x^2 + y^2$. We solve

$$\begin{cases} x^2 + y^2 = 1 \\ 3 = \lambda 2x \\ 4 = \lambda 2y \end{cases}$$

simultaneously. We see that λ is not zero (because $3 \neq 0$). It follows that $x = \frac{3}{2\lambda}$, $y = \frac{2}{\lambda}$, and

$$\left(\frac{3}{2\lambda}\right)^2 + \left(\frac{2}{\lambda}\right)^2 = 1$$

$$9 + 16 = 4\lambda^2$$

$\lambda = \pm 5/2$. The extreme points of f on $x^2 + y^2 = 1$ occur when (x, y) equals $(3/5, 4/5)$ and $(-3/5, -4/5)$. We calculate $f(3/5, 4/5) = 5$ and $f(-3/5, -4/5) = -5$. We conclude that

The maximum of f on $x^2 + y^2 = 1$ is 5 and $f(3/5, 4/5) = 5$ and the minimum of f on $x^2 + y^2 = 1$ is -5 and $f(-3/5, -4/5) = -5$.

(5) **Let** $f(x,y) = 2x^3y^4 \sin(3x^2y^5)$**. Find** $\frac{\partial f}{\partial y}$**.**

$$\frac{\partial f}{\partial y} = \boxed{2x^3y^4(15x^2y^4)\cos(3x^2y^5) + 8x^3y^3\sin(3x^2y^5).}$$