Math 241, Exam 3, Fall, 2017 11:40 class Solutions

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Tuesday.

No Calculators, Cell phones, computers, notes, etc.

(1) **Graph, name, describe the set of points in 3-space which satisfy**

$$x^2 + y^2 - z^2 = 1.$$

See the picture page.

(2) **The position vector of an object at time t is $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$, for some functions $x = x(t)$ and $y = y(t)$. Suppose**

$$\vec{r}''(t) = -4\vec{j} \text{ for all } t, \quad \vec{r}'(0) = 2\vec{i} + 3\vec{j}, \quad \text{and} \quad \vec{r}(0) = 0.$$

Find the x-coordinate of the object when the y-coordinate is 1.

Integrate to learn $\vec{r}'(t) = -4t\vec{j} + \vec{c_1}$. Plug in $t = 0$ to see

$$2\vec{i} + 3\vec{j} = \vec{r}'(0) = \vec{c_1};$$

thus,

$$\vec{r}'(t) = 2\vec{i} + (3 - 4t)\vec{j}.$$

Integrate again

$$\vec{r}(t) = 2t\vec{i} + (3t - 2t^2)\vec{j}.$$

The y-coordinate is 1, when $3t - 2t^2 = 1$; so

$$0 = 2t^2 - 3t + 1 = (2t - 1)(t - 1)$$

and $t = 1/2$ or $t = 1$.

$\boxed{\text{When } t = 1/2, \text{ the } x\text{-coordinate is } 1. \text{ When } t = 1, \text{ the } x\text{-coordinate is } 2.}$

(3) **Find the equation of the plane tangent to $z = x^2 + y^2$ when $x = 1$ and $y = 3$.**

Gradients are perpendicular to level sets. We view the paraboloid as the level set $0 = -z + x^2 + y^2$. The gradient of the right side evaluated at the point $(1, 3, 10)$ is

$$(2x\vec{\boldsymbol{i}} + 2y\vec{\boldsymbol{j}} - \vec{\boldsymbol{k}})|_{(1,3,10)} = 2\vec{\boldsymbol{i}} + 6\vec{\boldsymbol{j}} - \vec{\boldsymbol{k}}.$$

The tangent plane is

$$\boxed{2(x-1) + 6(y-3) - (z-10) = 0}.$$

(4) **Find the absolute maximum and minimum values of**

$$f(x, y) = 2 + 2x + 4y - x^2 - y^2$$

on the triangular region in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

We drew the region on the picture page.

- We first look for interior points where f_x and f_y both vanish. We compute $f_x = 2 - 2x$ and $f_y = 4 - 2y$. We see that the only point where $f_x = 0$ and $f_y = 0$ is $\underline{(1, 2)}$.
- We find points on $x = 0$ where the derivative of (f restricted to $x = 0$) vanishes. We are interested in $f(y) = 2 + 4y - y^2$ with $0 \le y \le 9$. We compute $f'(y) = 4 - 2y$. We consider $\underline{(0, 2)}$.
- We find points on $y = 9 - x$ where the derivative of (f restricted to $y = 9 - x$) vanishes. We are interested in

$$f(x) = 2 + 2x + 4(9 - x) - x^2 - (9 - x)^2$$

  with $0 \le x \le 9$. We compute $f'(x) = 2 - 4 - 2x + 2(9 - x) = 16 - 4x$. We consider $\underline{(4, 5)}$.
- We find points on $y = 0$ where the derivative of (f restricted to $y = 0$) vanishes. We are interested in $f(x) = 2 + 2x - x^2$ for $0 \le x \le 9$. We compute $f'(x) = 2 - 2x$. We consider $\underline{(1, 0)}$.
- We also consider the corner points $\underline{(0, 0)}$, $\underline{(0, 9)}$, $\underline{(9, 0)}$.

The maximum and the minimum of f occur at one of the underlined points. We compute f at each of these points:

$$\begin{aligned}
f(1, 2) &= 7 \\
f(0, 2) &= 6 \\
f(4, 5) &- 11 \\
f(1, 0) &= 3 \\
f(0, 0) &= 2 \\
f(0, 9) &= -43 \\
f(9, 0) &= -61
\end{aligned}$$

Thus,

The maximum of f on the given region is 7 and $f(1, 2) = 7$. The minimum of f on the given region is -61 and $f(9, 0) = -61$.

(5) **Let** $f(x, y) = 2x^3y^4 \sin(3x^2y^5)$**. Find** $\frac{\partial f}{\partial x}$**.**

We compute

$$\boxed{\frac{\partial f}{\partial x} = 2x^3y^4(6xy^5)\cos(3x^2y^5) + 6x^2y^4\sin(3x^2y^5).}$$