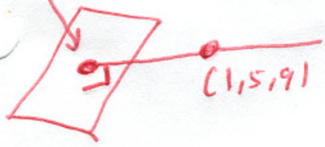
Find me

9. Find the point on $x + 2y + 3z = 10$ which is closest to $(1, 5, 9)$.



The point is

$(-1, 1, 3)$

The line is $\frac{x-1}{1} = \frac{y-5}{2} = \frac{z-9}{3}$

or $x = 1+t$
$y = 5+2t$
$z = 9+3t$

The line hits the plane when

$(1+t) + 2(5+2t) + 3(9+3t) = 10$

$t + 4t + 9t + 1 + 10 + 27 = 10$

$14t = -28$

$t = -2$

Check $(-1,1,3)$ is on the plane

$-1+2+9 = 10$ ✓

The distance from $(1,5,9)$ to the plane is

$$\frac{|1+10+27-10|}{\sqrt{1+4+9}} = \frac{28}{\sqrt{14}} = 2\sqrt{14}$$

the distance from $(1,5,9)$ to $(-1,1,3)$ is $\sqrt{4+16+36} = \sqrt{56} = \sqrt{4 \cdot 14} = 2\sqrt{14}$

10. Find the length of the curve whose position vector is

$$\vec{r}(t) = t^2\,\vec{i} - 2t^3\,\vec{j} + 6t^3\,\vec{k},$$

for $0 \le t \le 1$.

$$\text{length} = \int_0^1 \|\vec{r}\,'(t)\|\,dt = \int_0^1 \|2t\vec{i} - 6t^2\vec{j} + 18t^2\vec{k}\|\,dt = \int_0^1 \|2t(\vec{i} - 3t\vec{j} + 9t\vec{k})\|\,dt$$

$$= \int_0^1 2t\sqrt{1+9t^2+81t^2}\,dt = \int_0^1 2t\sqrt{1+90t^2}\,dt = \int_1^{91} \frac{2}{180} u^{\frac{1}{2}}\,du$$

$u = 1+90t^2$
$du = 180t\,dt$
when $t=0$ $u=1$
when $t=1$ $u=91$
$\frac{du}{180} = t\,dt$

$$= \frac{1}{90}\,\frac{2}{3}\,u^{\frac{3}{2}}\Big]_1^{91}$$

$$= \frac{2}{270}\left((91)^{\frac{3}{2}} - 1\right)$$