5. Find the area of the triangle with vertices $P = (1, 2, 4)$, $Q = (2, 1, 2)$, and $R = (2, 4, 6)$.

area $= \frac{1}{2} \| \vec{PQ} \times \vec{PR} \|$

$= \frac{1}{2} \left\| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -2 \\ 1 & 2 & 2 \end{vmatrix} \right\|$

$= \frac{1}{2} \| (-2+4)\vec{i} - (2+2)\vec{j} + (2+1)\vec{k} \|$

$= \frac{1}{2} \| 2\vec{i} - 4\vec{j} + 3\vec{k} \|$

$= \frac{1}{2} \sqrt{4 + 16 + 9}$

6. Find the intersection of the two lines:

$$\frac{x-8}{3} = \frac{y-7}{4} = \frac{z-6}{1} \quad \text{and} \quad \frac{x-5}{1} = \frac{y-5}{2} = \frac{z-13}{3}.$$

Check your answer.

I walk on the line on the left.
At time t I stand on

$x = 8 + 3t$
$y = 7 + 4t$
$z = 6 + t$

When do I stand on the other line?

need $\frac{8+3t-5}{1} = \frac{7+4t-5}{2} = \frac{6+t-13}{3}$

I need

$\begin{cases} 3t+3 = \frac{2+4t}{2} \\ 3t+3 = \frac{t-7}{3} \end{cases}$

$\begin{cases} 3t+3 = 1+2t \\ 9t+9 = t-7 \end{cases}$

$\begin{cases} t = -2 \\ 8t = -16 \end{cases}$

so $t = -2$

The point is

$x = 8 + 3(-2) = 2$
$y = 7 + 4(-2) = -1$
$z = 6 + (-2) = 4$

$(2, -1, 4)$

check:

$(2, -1, 4)$ is on the left line because:

$\frac{-6}{3} = \frac{-8}{4} = \frac{-2}{1}$ ✓

rt line:

$\frac{-3}{1} = \frac{-6}{2} = \frac{-9}{3}$ ✓