

3. Find the equations of the line through the points $P = (2, -3, 4)$ and $Q = (3, 4, 6)$. Check your answer.

$$\vec{PQ} = \vec{i} + 7\vec{j} + 2\vec{k}$$

$$\boxed{\frac{x-2}{1} = \frac{y+3}{+7} = \frac{z-4}{2}}$$

check: P satisfies these equations because

$$\frac{2-2}{1} = \frac{-3+3}{+7} = \frac{4-4}{2} \quad \text{all are } 0$$

Q satisfies these equations because

$$\frac{3-2}{1} = \frac{4+3}{7} = \frac{6-4}{2} \quad \text{all are } 1$$

4. Find the equation of the plane through the points $P = (1, 1, 2)$, $Q = (2, 3, 6)$, and $R = (-1, -1, 0)$. Check your answer.

$$\vec{PQ} = \vec{i} + 2\vec{j} + 4\vec{k}$$

$$\vec{PR} = -2\vec{i} - 2\vec{j} - 2\vec{k}$$

$$\begin{aligned} \vec{PQ} \times \vec{PR} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 4 \\ -2 & -2 & -2 \end{vmatrix} = (-4+8)\vec{i} - (-2+8)\vec{j} + (-2+4)\vec{k} \\ &= 4\vec{i} - 6\vec{j} + 2\vec{k} \\ &= 2(2\vec{i} - 3\vec{j} + \vec{k}) \end{aligned}$$

The plane through $(1, 1, 2) + 2\vec{i} - 3\vec{j} + \vec{k}$

$$\text{is } 2(x+1) - 3(y+1) + z = 0$$

$$2x - 3y + z + 2 - 3 = 0$$

check: $2x - 3y + z = 1$

P: $2 - 3 + 2 = 1 \checkmark$

Q: $4 - 9 + 6 = 1 \checkmark$

R: $-2 + 3 + 0 = 1 \checkmark$