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7. Find the length of the curve $\overrightarrow{r}(t) = \sqrt{6}t^2\overrightarrow{i} + \frac{2}{3}t^3\overrightarrow{j} + 6t\overrightarrow{k}$ for $3 \leq t \leq 6$.

$$\int_3^6 \|\vec{r}\,'(t)\|\,dt = \int_3^6 \|2\sqrt{6}\,t\,\vec{i} + 2t^2\vec{j} + 6\vec{k}\|\,dt = \int_3^6 \sqrt{24t^2 + 4t^4 + 36}\,dt$$

$$= \int_3^6 2\sqrt{t^4+6t^2+9}\,dt = \int_3^6 2\sqrt{(t^2+3)^2}\,dt = 2\int_3^6 t^2+3\,dt$$

$$= \left(\frac{t^3}{3} + 3t\right)2\Big]_3^6 = 2\,(36\cdot 2 + 18 - 9 - 9)$$

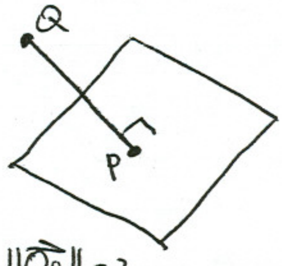
$$\|$$

$$4\cdot 36$$

8. Find the point on $2x + y + 2z = 4$ which is closest to $(1, 2, 3)$. $= Q$

The distance from the point to the plane is $\dfrac{|2+2+6-4|}{\sqrt{4+1+4}} = \dfrac{6}{3} = 2.$

So we want $\underset{(x,y,z)}{\overset{\|}{P}}$ on the plane with $\overrightarrow{QP} = \pm\frac{2}{3}(2\vec{i} + \vec{j} + 2\vec{k})$



$\|\overrightarrow{QP}\| = 2$

$\overrightarrow{QP} \,\|\, 2\vec{i}+\vec{j}+2\vec{k}$

so either $x = 1 + \frac{4}{3}$ or $x = 1 - \frac{4}{3} = -\frac{1}{3}$

$y = 2 + \frac{2}{3}$ $\quad y = 2 - \frac{2}{3} = \frac{4}{3}$

$z = 3 + \frac{4}{3}$ $\quad z = 3 - \frac{4}{3} = \frac{5}{3}$

$\uparrow$

on the plane

$$\left(-\frac{1}{3}, \frac{4}{3}, \frac{5}{3}\right)$$