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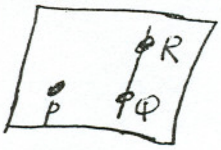
7. Find the equation of the plane which contains the point $\overset{P}{(2,1,3)}$ and the line

$$\begin{cases} x = 1 + t \\ y = -1 + 8t \\ z = 3 - 2t. \end{cases}$$

CHECK YOUR ANSWER!



P is on the plane
$-4+1+9=6$ ✓

$Q: -2-1+9=6$ ✓

$R: -4+7+3=6$ ✓

$P=(2,1,3)$

$Q=(1,-1,3)$

$R=(2,7,1)$

$\vec{PQ} \times \vec{QR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 0 \\ 1 & 8 & -2 \end{vmatrix} = -4\vec{i} + 2\vec{j} + 6\vec{k} = 2(-2\vec{i} + \vec{j} + 3\vec{k})$

$-2(x-2) + (y-1) + 3(z-3) = 0$

$-2x + y + 3z + 4 - 1 - 9 = 0$

$-2x + y + 3z = 6$

8. Find the length of the curve parameterized by $\vec{r}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t^{\frac{3}{2}}\,\vec{k}$, for $0 \le t \le \frac{20}{3}$.

$\text{length} = \int_0^{\frac{20}{3}} \|\vec{r}\,'(t)\|\,dt = \int_0^{\frac{20}{3}} \|-\sin t\,\vec{i} + \cos t\,\vec{j} + \frac{3}{2}t^{\frac{1}{2}}\vec{k}\|\,dt = \int_0^{\frac{20}{3}} \sqrt{\sin^2 t + \cos^2 t + \frac{9}{4}t}\,dt$

$= \int_0^{\frac{20}{3}} \sqrt{1 + \frac{9}{4}t}\,dt = \frac{4}{9}\cdot\frac{2}{3}\left(1 + \frac{9}{4}t\right)^{\frac{3}{2}}\Big]_0^{\frac{20}{3}}$

$= \frac{8}{27}\left(\left(1 + \frac{\cancel{9}^{3}}{\cancel{4}}\cdot\frac{\cancel{20}^{5}}{\cancel{3}}\right)^{\frac{3}{2}} - 1\right)$

$= \frac{8}{27}\left((16)^{\frac{3}{2}} - 1\right)$

$= \frac{8}{27}(64 - 1)$

$= \frac{8}{27}(63)$

$= \frac{8}{3}7 = \frac{56}{3}$