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5. What are the equations of the line tangent to the curve parameterized by
$\vec{r}(t) = 3t^3 \vec{i} + 2t^2 \vec{j} + 4t^5 \vec{k}$ at $t = -1$?

at $t=-1$ the position is $(-3, 2, -4)$

$\vec{r}'(t) = 9t^2 \vec{i} + 4t \vec{j} + 20t^4 \vec{k}$

$\vec{r}'(-1) = 9\vec{i} - 4\vec{j} + 20\vec{k}$

tan line $\begin{cases} x = -3 + 9t \\ y = 2 - 4t \\ z = -4 + 20t \end{cases}$

6. (There is no partial credit for this problem. Make sure your answer is correct.) Let $\vec{a} = -\vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = -3\vec{i} + 5\vec{j} + 5\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$.

$\vec{u} = \text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{3+10+15}{1+4+9}\vec{a} = \frac{28}{14}\vec{a} = 2\vec{a}$

$\boxed{\vec{u} = -2\vec{i} + 4\vec{j} + 6\vec{k}}$

$\vec{v} = \vec{b} - \vec{u} = \langle -3, 5, 5 \rangle - \langle -2, 4, 6 \rangle = \boxed{\langle -1, 1, -1 \rangle = \vec{v}}$

check $\vec{u}$ is $\parallel \vec{a}$ ✓

$\vec{u} + \vec{v} = -3\vec{i} + 5\vec{j} + 5\vec{k}$ ✓

$\vec{v}\cdot\vec{a} = 1 + 2 - 3 = 0$ ✓