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7. Find the length of the curve
$\vec{r}(t) = \sqrt{6}t^2\,\vec{i} + \frac{2}{3}t^3\,\vec{j} + 6t\,\vec{k}$ for $2 \le t \le 6$.

arc length $= \int_{\text{beg.}}^{\text{end}} \|\vec{r}'(t)\|\,dt = \int_2^6 \|2\sqrt{6}t\,\vec{i} + 2t^2\,\vec{j} + 6\,\vec{k}\|\,dt = \int_2^6 \sqrt{24t^2 + 4t^4 + 36}$

$= 2\int_2^6 \sqrt{t^4 + 6t^2 + 9}\,dt = 2\int_2^6 \sqrt{(t^2+3)^2}\,dt = 2\int_2^6 t^2 + 3\,dt$

$= 2\left(\frac{t^3}{3} + 3t\right)\Big]_2^6 = 2\left(\frac{6^3}{3} + 18 - \left(\frac{8}{3} + 6\right)\right)$

8. What are the equations of the line tangent to the curve
$\vec{r}(t) = (3t^2 + 2)\,\vec{i} + 4t\,\vec{j} + (4t^2 + 2t)\,\vec{k}$ at $t = 1$?

The position of the object at $t=1$ is $(5, 4, 6)$

$\vec{r}'(t) = 6t\,\vec{i} + 4\,\vec{j} + (8t+2)\,\vec{k}$

$\vec{r}'(1) = 6\,\vec{i} + 4\,\vec{j} + 10\,\vec{k}$

The line through $(5,4,6)$ $\parallel$ $\vec{r}'(1)$ is $\frac{x-5}{6} = \frac{y-4}{4} = \frac{z-6}{10}$