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7. Find the length of the curve

$$\vec{r}(t) = \sqrt{6}t^2 \vec{i} + \frac{2}{3}t^3 \vec{j} + 6t \vec{k} \text{ for } 3 \leq t \leq 6.$$

$$\vec{r}'(t) = 2\sqrt{6}t \vec{i} + 2t^2 \vec{j} + 6 \vec{k}$$

$$\|\vec{r}'(t)\| = \sqrt{24t^2 + 4t^4 + 36} = 2\sqrt{6t^2 + t^4 + 9} = 2\sqrt{(t^2+3)^2} = 2(t^2+3)$$

$$\text{length} = 2 \int_3^6 (t^2+3) dt = 2 \left[\frac{t^3}{3} + 3t \right]_3^6$$

$$= 2 \left(\frac{6^3}{3} + 18 - (9+9) \right)$$

8. What are the equations of the line tangent to the curve

$$\vec{r}(t) = (3t^2 + 1) \vec{i} + 6t \vec{j} + (4t^3 + 2t) \vec{k} \text{ at } t = 1?$$

$$\vec{r}(1) = 4 \vec{i} + 6 \vec{j} + 6 \vec{k}$$

$$\text{so pt is } (4, 6, 6)$$

$$\vec{r}'(t) \text{ is } 6t \vec{i} + 6 \vec{j} + (12t^2 + 2) \vec{k}$$

$$\vec{r}'(1) = 6 \vec{i} + 6 \vec{j} + 14 \vec{k} \quad \leftarrow \text{ // vector}$$

$$\frac{x-4}{6} = \frac{y-6}{6} = \frac{z-6}{14}$$