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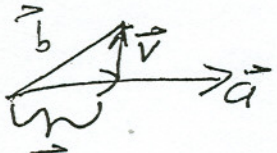
7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 3\vec{i} + 7\vec{j} + 13\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$.



$$\vec{u} = \text{proj}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{3+14+39}{1+4+9}\vec{a} = \frac{56}{14}\vec{a}$$

$$= 4\vec{a} = 4\vec{i} + 8\vec{j} + 12\vec{k} = \vec{u}$$

$$\vec{v} = \vec{b} - \vec{u} = 3\vec{i} + 7\vec{j} + 13\vec{k} - (4\vec{i} + 8\vec{j} + 12\vec{k})$$

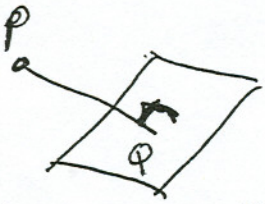
$$= -\vec{i} - \vec{j} + \vec{k} = \vec{v}$$

✓: $\vec{u} \parallel \vec{a}$ ✓

$\vec{u} + \vec{v} = \vec{b}$ ✓

$\vec{v} \perp \vec{a}$: $\vec{v}\cdot\vec{a} = -1 - 2 + 3 = 0$ ✓

8. Find the point on $2x - y - 3z = 19$ which is closest to $(1, 2, 3)$.



The line connecting P and Q is

$$\frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{-3}$$

Or $x = 1 + 2t$
$y = 2 - t$
$z = 3 - 3t$

If I walk on this line then I hit the plane when

$2(1+2t) - (2-t) - 3(3-3t) = 19$

$2 + 4t - 2 + t - 9 + 9t = 19$

$14t = 28$

so $t = 2$

So Q is $(5, 0, -3)$

P

- Q is on the plane:

$10 - 0 + 9 = 19$ ✓

The distance from P to Q is

$\sqrt{4^2 + 2^2 + 6^2} = \sqrt{56} = 2\sqrt{14}$

The distance from P to the plane is ✓

$$\frac{|2 - 2 - 9 - 19|}{\sqrt{4+1+9}} = \frac{28}{\sqrt{14}} = \frac{28\sqrt{14}}{14} = 2\sqrt{14}$$