B
52

5. Find the point of intersection of the following lines. **CHECK YOUR ANSWER!**

$$\frac{x+5}{-1} = \frac{y-10}{4} = \frac{z+3}{-1} \quad \text{and} \quad \frac{x}{1} = \frac{y-8}{2} = \frac{z-8}{3}$$

Walk along the first line.
At time t you stand on
$x = -t-5$
$y = 4t+10$
$z = -t-3$
What time works the equations of the second line?

$$\frac{-t-5}{1} = \frac{4t+10-8}{2} = \frac{-t-3-8}{3}$$

$$\left.\begin{array}{l} -t-5 = 2t+1 \\ -t-5 = \frac{-t-11}{3} \end{array}\right\}$$

$$\left.\begin{array}{l} -6 = 3t \\ -3t-15 = -t-11 \end{array}\right\}$$

$$\begin{cases} t = -2 \\ -4 = 2t \end{cases}$$

so $t = -2$

$(-3, 2, -1)$

6. Find the length of the curve parameterized by $\overrightarrow{r}(t) = \sqrt{6}t^2\,\overrightarrow{i} + \frac{2}{3}t^3\,\overrightarrow{j} + 6t\,\overrightarrow{k}$ for $3 \le t \le 6$.

$$\text{length} = \int_3^6 \|\vec{r}\,'(t)\|\,dt = \int_3^6 \|2\sqrt{6}t\,\vec{i} + 2t^2\vec{j} + 6\vec{k}\|\,dt = \int_3^6 \sqrt{24t^2 + 4t^4 + 36}\,dt$$

$$= \int_3^6 \sqrt{(2t^2+6)^2}\,dt = \int_3^6 (2t^2+6)\,dt = \frac{2}{3}t^3 + 6t\Big]_3^6$$

$$= \frac{2}{3}(6)^3 + 36 - \frac{2}{3}(3)^3 - 18$$