Math 241, Exam 2, Spring, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) **Find an equation for the plane through the points $P_1 = (1,1,1)$, $P_2 = (2,4,5)$, and $P_3 = (3,-1,2)$. Check your answer. Make sure it is correct.**

We calculate $\overrightarrow{P_1P_2} = \vec{\boldsymbol{i}} + 3\vec{\boldsymbol{j}} + 4\vec{\boldsymbol{k}}$ and $\overrightarrow{P_1P_3} = 2\vec{\boldsymbol{i}} - 2\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$; thus,

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 1 & 3 & 4 \\ 2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} \vec{\boldsymbol{i}} - \begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} \vec{\boldsymbol{j}} + \begin{vmatrix} 1 & 3 \\ 2 & -2 \end{vmatrix} \vec{\boldsymbol{k}}$$

$$= 11\vec{\boldsymbol{i}} + 7\vec{\boldsymbol{j}} - 8\vec{\boldsymbol{k}}.$$

The plane through $(1,1,1)$ perpendicular to $11\vec{\boldsymbol{i}} + 7\vec{\boldsymbol{j}} - 8\vec{\boldsymbol{k}}$ is

$$11(x-1) + 7(y-1) - 8(z-1) = 0$$

$$\boxed{11x + 7y - 8z = 10}.$$

Check. Plug $(1,1,1)$ into the proposed answer to get

$$11 + 7 - 8 = 10\checkmark.$$

Plug $(2,4,5)$ into the proposed answer to get

$$11(2) + 7(4) - 8(5) = 10\checkmark.$$

Plug $(3,-1,2)$ into the proposed answer to get

$$11(3) + 7(-1) - 8(2) = 10\checkmark.$$

(2) **Find $\frac{\partial}{\partial x}\left(x^2 y \cos^2(3x^3 y + e^{xy})\right)$.**

We compute that $\frac{\partial}{\partial x}\left(x^2 y \cos^2(3x^3 y + e^{xy})\right)$

$$= \boxed{x^2 y(2)\cos(3x^3 y + e^{xy})(-\sin(3x^3 y + e^{xy}))(9x^2 y + ye^{xy}) + 2xy\cos^2(3x^3 y + e^{xy}).}$$

(3) **An object moves in three space. The position vector of the object at time t is $\vec{r}(t) = 2t^2\vec{i} + 3t^3\vec{j} + 4t^4\vec{k}$. Find parametric equations for the line tangent to the path of the object when the object stands at the point $(2, 3, 4)$.**

The object stands at $(2, 3, 4)$ at $t = 1$. The tangent line is parallel to the vector $\vec{r}\,'(1)$. We compute $\vec{r}\,'(t) = 4t\vec{i} + 9t^2\vec{j} + 16t^3\vec{k}$. Thus $\vec{r}\,'(1) = 4\vec{i} + 9\vec{j} + 16\vec{k}$. The line through $(2, 3, 4)$ parallel to $4\vec{i} + 9\vec{j} + 16\vec{k}$ is

$$\boxed{\begin{cases} x = 2 + 4t \\ y = 3 + 9t \\ z = 4 + 16t. \end{cases}}$$

(4) **An object moves in three space. The position vector of the object at time t is $\vec{r}(t) = \cos(3t)\vec{i} + \sin(3t)\vec{j} + t\vec{k}$. How far does the object travel between $t = 0$ and $t = 2\pi$?**

The distance travelled is equal to

$$\int_0^{2\pi} |\vec{r}\,'(t)|dt = \int_0^{2\pi} |-3\sin(3t)\vec{i} + 3\cos(3t)\vec{j} + \vec{k}|dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2(3t) + 9\cos^2(3t) + 1}|dt = \int_0^{2\pi} \sqrt{10}dt = \boxed{2\sqrt{10}\pi}.$$

(5) **Name, graph, and describe the set of points in 3-space that satisfy the equation $z^2 = x^2 + y^2$.**

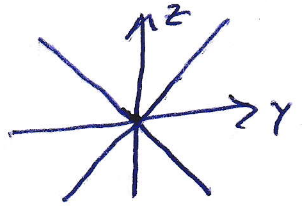
See the next page.

⑤ $z^2 = x^2 + y^2$

when $z = 0$ the equation is $0 = x^2 + y^2$. The graph of this equation is a Point.

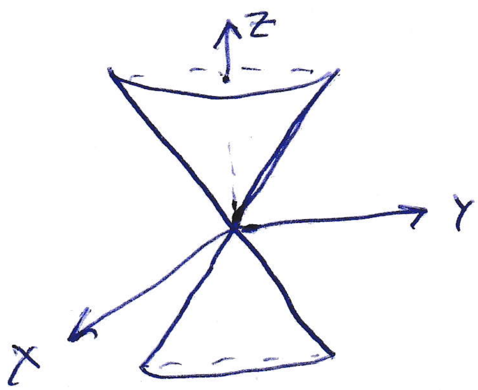
when z is a non-zero Constant, the equation is $constant^2 = x^2 + y^2$, The graph of this equation is a circle

When $x = 0$ the equation is $z^2 = y^2$, The graph of this equation is 2 straight lines



when $Y = 0$ the equation is $z^2 = x^2$. The graph of this equation is also 2 straight lines

The graph of $z^2 = x^2 + y^2$ is a cone



Take 2 traffic cones and join them tip to tip.