Math 241, Exam 2, Spring, 2019

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Thursday.

No Calculators, Cell phones, computers, notes, etc.

(1) **Let $f(x,y) = x\sqrt{x\cos y + 3x^2}$. Find $\frac{\partial f}{dx}$ and $\frac{\partial f}{dy}$.**

$$\boxed{\frac{\partial f}{dx} = x\frac{\cos y + 6x}{2\sqrt{x\cos y + 3x^2}} + \sqrt{x\cos y + 3x^2}}$$

$$\boxed{\frac{\partial f}{dy} = \frac{-x^2 \sin y}{2\sqrt{x\cos y + 3x^2}}}$$

(2) **Describe and graph $x^2 + y^2 - z^2 = 1$ in three-space. What is the name of this object?**

See the picture page of the solution.

(3) **Find the point of intersection of the two lines**

$$\begin{cases} x = 3 - t \\ y = 3 + 2t \\ z = 10 + 5t \end{cases} \quad \textbf{and} \quad \begin{cases} x = 6 + s \\ y = 5 + 2s \\ z = 11 + 3s. \end{cases}$$

The equations on the left give the position of object one at time t. The equations on the right give the position of object two at time t. We look for a time t_0 and a time s_0 with the position of object one at time t_0 equal to the position of object two at time s_0.

We solve

$$\begin{cases} 3 - t_0 = 6 + s_0 \\ 3 + 2t_0 = 5 + 2s_0 \\ 10 + 5t_0 = 11 + 3s_0 \end{cases}$$

simultaneously. We solve

$$\begin{cases} -3 - t_0 = s_0 \\ 3 + 2t_0 = 5 + 2(-3 - t_0) \\ 10 + 5t_0 = 11 + 3(-3 - t_0) \end{cases}$$

simultaneously. We solve

$$\begin{cases} -3 - t_0 = s_0 \\ 4t_0 = 5 + 2(-3) - 3 \\ 8t_0 = 11 + 3(-3) - 10 \end{cases}$$

simultaneously. We solve

$$\begin{cases} -3 - t_0 = s_0 \\ 4t_0 = -4 \\ 8t_0 = -8 \end{cases}$$

simultaneously. So, $t_0 = -1$ and $s_0 = -2$. The point is $\boxed{(4, 1, 5).}$

(4) **Find the length of the graph for $y = x^{3/2}$ on the closed interval $0 \leq x \leq 4$.**

The curve is $\overrightarrow{\boldsymbol{r}}(t) = t\,\overrightarrow{\boldsymbol{i}} + t^{3/2}\overrightarrow{\boldsymbol{j}}$. The length of the curve is

$$\int_0^4 |\overrightarrow{\boldsymbol{r}}'(t)|dt = \int_0^4 |\overrightarrow{\boldsymbol{i}} + (3/2)t^{1/2}\overrightarrow{\boldsymbol{j}}|dt = \int_0^4 \sqrt{1^2 + ((3/2)t^{1/2})^2}dt$$

$$= \int_0^4 \sqrt{1 + (9/4)t}dt = (2/3)(4/9)(1 + (9/4)t)^{3/2}|_0^4 = \boxed{(8/27)(10^{3/2} - 1)}$$

(5) **An object starts at the origin with velocity $4\overrightarrow{\boldsymbol{i}} + 8\overrightarrow{\boldsymbol{j}}$. The acceleration of the object at time t is $\overrightarrow{\boldsymbol{r}}''(t) = 2e^t\overrightarrow{\boldsymbol{i}} + 16e^{2t}\overrightarrow{\boldsymbol{j}}$. What is the x-coordinate of the object when the y=coordinate is 12?**

Integrate to learn that $\overrightarrow{\boldsymbol{r}}'(t) = 2e^t\overrightarrow{\boldsymbol{i}} + 8e^{2t}\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{c}}_1$. Plug in $t = 0$ to learn

$$4\overrightarrow{\boldsymbol{i}} + 8\overrightarrow{\boldsymbol{j}} = \overrightarrow{\boldsymbol{r}}'(0) = 2\overrightarrow{\boldsymbol{i}} + 8\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{c}}_1.$$

So, $\overrightarrow{\boldsymbol{c}}_1 = 2\overrightarrow{\boldsymbol{i}}$,

$$\overrightarrow{\boldsymbol{r}}'(t) = (2e^t + 2)\overrightarrow{\boldsymbol{i}} + 8e^{2t}\overrightarrow{\boldsymbol{j}}.$$

Integrate again to learn

$$\overrightarrow{\boldsymbol{r}}(t) = (2e^t + 2t)\overrightarrow{\boldsymbol{i}} + 4e^{2t}\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{c}}_2.$$

Plug in $t = 0$ to learn

$$0 = \vec{\boldsymbol{r}}(0) = 2\vec{\boldsymbol{i}} + 4\vec{\boldsymbol{j}} + \vec{\boldsymbol{c}}_2.$$

So, $\vec{\boldsymbol{c}}_2 = -2\vec{\boldsymbol{i}} - 4\vec{\boldsymbol{j}}$ and

$$\vec{\boldsymbol{r}}(t) = (2e^t + 2t - 2)\vec{\boldsymbol{i}} + (4e^{2t} - 4)\vec{\boldsymbol{j}}.$$

The y-coordinate of the object is 12 when $4e^{2t} - 4 = 12$; thus, $4e^{2t} = 16$;

$$e^{2t} = 4$$

$$2t = \ln 4$$

$$t = (\ln 4)/2 = (2\ln 2)/2 = \ln 2.$$

The x-coordinate of the object is $2e^{\ln 2} + 2\ln 2 - 2 = 4 + \ln 4 - 2 = \boxed{2 + \ln 4}$ when the y-coordinate of the object is 12.