**Math 241, Exam 2, Fall, 2017 1:15 class**

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Thrusday.

**No Calculators, Cell phones, computers, notes, etc.**

(1) **Express $\vec{v} = \vec{i} + 2\vec{j}$ as the sum of a vector parallel to $\vec{b} = 5\vec{i} + 4\vec{j}$ plus a vector perpendicular to $\vec{b}$. Check your answer. Make sure it is correct.**

Observe that $\vec{v} = \text{proj}_{\vec{b}}\,\vec{v} + (\vec{v} - \text{proj}_{\vec{b}}\,\vec{v})$ with $\text{proj}_{\vec{b}}\,\vec{v}$ parallel to $\vec{b}$ and $(\vec{v} - \text{proj}_{\vec{b}}\,\vec{v})$ perpendicular to $\vec{b}$.

We compute

$$\text{proj}_{\vec{b}}\,\vec{v} = \frac{\vec{b}\cdot\vec{v}}{\vec{b}\cdot\vec{b}}\vec{b} = \frac{(5\vec{i}+4\vec{j})\cdot(\vec{i}+2\vec{j})}{(5\vec{i}+4\vec{j})\cdot(5\vec{i}+4\vec{j})}(5\vec{i}+4\vec{j}) = \frac{13}{41}(5\vec{i}+4\vec{j}).$$

It follows that

$$(\vec{v} - \text{proj}_{\vec{b}}\,\vec{v}) = \vec{i} + 2\vec{j} - \frac{13}{41}(5\vec{i}+4\vec{j}) = -\frac{24}{41}\vec{i} + \frac{30}{41}\vec{j}.$$

Thus $\vec{v} = \frac{13}{41}(5\vec{i}+4\vec{j}) + (-\frac{24}{41}\vec{i} + \frac{30}{41}\vec{j})$ with $\frac{13}{41}(5\vec{i}+4\vec{j})$ parallel to $\vec{b}$ and $-\frac{24}{41}\vec{i} + \frac{30}{41}\vec{j}$ perpendicular to $\vec{b}$.

(2) **Let $f(x,y) = 3x^2\sin(3y) + 7y\cos(2x)$. Find $\frac{\partial f}{\partial y}$.**

$$\boxed{\frac{\partial f}{\partial y} = 9x^2\cos(3y) + 7\cos(2x)}$$

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(3) **Find the point on the plane $x + 2y + 3z = 2$ which is closest to the point $(2,1,3)$.**

The line through $(2,1,3)$ which is perpendicular to the plane is

$$x = 2 + t, \quad y = 1 + 2t, \quad z = 3 + 3t.$$

The line and the plane meet when

$$(2+t)+2(1+2t)+3(3+3t)=2.$$

$$14t=-11.$$

The line and the plane meet at $t=-11/14$. The point of intersection is

$$(2-11/14, 1-22/14, 3-33/14)=\boxed{(17/14,-8/14,9/14)}.$$

**Check.**The point is on the plane because

$$(1/14)(17-16+27)=2.$$

The vector

$$\overrightarrow{(2,1,3)(17/14,-8/14,9/14)}=(1/14)((17-28)\vec{\boldsymbol{i}}+(-8-14)\vec{\boldsymbol{j}}+(9-42)\vec{\boldsymbol{k}})$$

$$=(-11/14)(\vec{\boldsymbol{i}}+2\vec{\boldsymbol{j}}+3\vec{\boldsymbol{k}})$$

is perpendicular to the plane.

(4) **Describe and graph the set of all points in three space which satisfy the equation $x^2+z^2=y^2$.**

The graph is a cone with the $y$-axis in its center. See the picture.

(5) **The position vector of an object at time $t$ is $\vec{\boldsymbol{r}}(t)=\cos(2t)\vec{\boldsymbol{i}}-\sin(2t)\vec{\boldsymbol{j}}$.**

(a) **Eliminate the parameter and give the path of the object.**

Use the identity $\cos^2\theta+\sin^2\theta=1$ to see that $x^2+y^2=\cos^2(2t)+\sin^2(2t)=1$. The path of the object is $\boxed{x^2+y^2=1}$.

(b) **Is the object moving clock-wise or counter clock-wise? (Please explain.)**

At time zero the object is standing on $(0,0)$. At time $\frac{\pi}{4}$ the object is standing on $(0,-1)$. The object is moving $\boxed{\text{clock-wise}}$.

(c) **What is the speed of the object at time $t$?**

The speed at time $t$ is $|\vec{\boldsymbol{r}}'(t)|=|-2\sin(2t)\vec{\boldsymbol{i}}-2\cos(2t)\vec{\boldsymbol{j}}|=\sqrt{4\sin^2(2t)+4\cos^2(2t)}=\boxed{2}$

(d) **Draw the velocity vector $\vec{\boldsymbol{r}}'(\frac{\pi}{4})$ on a picture of the path of the object. Put the tail of the velocity vector on the position of the object at $t=\frac{\pi}{4}$.**

$\vec{\boldsymbol{r}}'(\frac{\pi}{4})=-2\sin(\frac{\pi}{2})\vec{\boldsymbol{i}}-2\cos(\frac{\pi}{2})\vec{\boldsymbol{j}}=\boxed{-2\vec{\boldsymbol{i}}}$. I drew the vector on the picture.