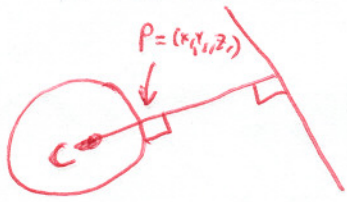
6. Find the equation of the plane which contains the point $(4, 2, 1)$ and is perpendicular to the vector $\vec{N} = 3\vec{i} - 2\vec{j} + 1\vec{k}$.

$3(x-4) - 2(y-2) + 1(z-1) = 0$

7. Find the point on $(x-1)^2 + (y-5)^2 + (z-9)^2 = 14$ which is closest to $x + 2y + 3z = 6$.



$C = (1, 5, 9)$

I want to find P

$\|\vec{CP}\| = \sqrt{14}$ which is the radius of the sphere

$\vec{CP} \perp$ the plane

so $\vec{CP} \parallel \vec{i} + 2\vec{j} + 3\vec{k} = \vec{N}$

So $\vec{CP} = \text{either } + \sqrt{14}\,\frac{\vec{N}}{\|\vec{N}\|}$ or $-\sqrt{14}\,\frac{\vec{N}}{\|\vec{N}\|}$

$\vec{CP} = \text{either } + \sqrt{14}\,\frac{\vec{i}+2\vec{j}+3\vec{k}}{\sqrt{1+4+9}}$ or $-\sqrt{14}\,\frac{\vec{i}+2\vec{j}+3\vec{k}}{\sqrt{1+4+9}}$

Of course $\vec{CP} = (x_1 - 1)\vec{i} + (y_1 - 5)\vec{j} + (z_1 - 9)\vec{k}$

So either

$(x_1 - 1)\vec{i} + (y_1 - 5)\vec{j} + (z_1 - 9)\vec{k} = \vec{i} + 2\vec{j} + 3\vec{k}$

or $(x_1 - 1)\vec{i} + (y_1 - 5)\vec{j} + (z_1 - 9)\vec{k} = -(\vec{i} + 2\vec{j} + 3\vec{k})$

So either $x_1 - 1 = 1$, $y_1 - 5 = 2$, $z_1 - 9 = 3$ or $x_1 - 1 = -1$, $y_1 - 5 = -2$, $z_1 - 9 = -3$

So $(x_1, y_1, z_1) = (2, 7, 12)$ or $(0, 3, 6)$

$(2, 7, 12)$: My distance to the plane is $\frac{|2 + 14 + 36 - 6|}{\sqrt{14}}$

$(0, 3, 6)$: my distance to the plane is $\frac{|0 + 6 + 18 - 6|}{\sqrt{14}}$

I am smaller

The point on $(x-1)^2 + (y-5)^2 + (z-9)^2 = 14$ which is closest to $x + 2y + 3z = 6$ is $(0, 3, 6)$