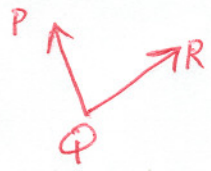
4. Consider the triangle with vertices $P = (1, 2, 4)$, $Q = (2, 1, 2)$, and $R = (2, 4, 6)$. Find the angle of this triangle at the vertex Q.



$\vec{QP} = (2,1,2) - (1,2,4) = -\vec{i} + \vec{j} + 2\vec{k}$

$\vec{QR} = (2,1,2) - (2,4,6) = 3\vec{j} + 4\vec{k}$

$\vec{QP} \cdot \vec{QR} = \|\vec{QP}\| \, \|\vec{QR}\| \cos\theta$

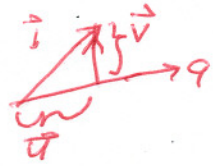
$\cos^{-1}\left(\dfrac{\vec{QP} \cdot \vec{QR}}{\|\vec{QP}\| \, \|\vec{QR}\|}\right) = \theta$

So $\theta = \cos^{-1}\left(\dfrac{3+8}{\sqrt{6}\ \sqrt{25}}\right)$

5. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{a} = 1\vec{i} + 2\vec{j} - 1\vec{k}$ and $\vec{b} = 2\vec{i} + 7\vec{j} - 2\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$. (Every number in the answer is an integer. If you have fractions, either you can rid of them or you have made a mistake.)

$\vec{u} = \text{proj}_{\vec{a}}\,\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}}\vec{a} = \dfrac{2+14+2}{1+4+1}\vec{a} = \dfrac{18}{6}\vec{a} = 3\vec{a} = 3\vec{i} + 6\vec{j} - 3\vec{k}$

$\vec{u}$



$\vec{v} = \vec{b} - \vec{u} = 2\vec{i} + 7\vec{j} - 2\vec{k} - (3\vec{i} + 6\vec{j} - 3\vec{k})$

$\vec{v} = -\vec{i} + \vec{j} + \vec{k}$

Check $\vec{u} \parallel \vec{a}$ ✓

$\vec{u} + \vec{v} = \vec{b}$ ✓

$\vec{v} \cdot \vec{a} = -1 + 2 - 1 = 0$ ✓