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9. Find the point on $2x + y + 2z = 1$ which is closest to $(2, 3, 3) = P$

The distance from P to the plane is $\frac{4+3+6-1}{\sqrt{4+1+4}} = \frac{12}{3} = 4$

Let P_0 be the point on the plane which is closest to P.
(x_0, y_0, z_0)

we know that $\overrightarrow{PP_0} = (x_0-2)\vec{i} + (y_0-3)\vec{j} + (z_0-3)\vec{k}$

also $\overrightarrow{PP_0} = \pm \frac{4}{3}(2\vec{i} + \vec{j} + 2\vec{k})$

so $x_0 = 2 + \frac{8}{3}$ $y_0 = 3 + \frac{4}{3}$, $z_0 = 3 + \frac{8}{3}$ or $x_0 = 2 - \frac{8}{3}$, $y_0 = 3 - \frac{4}{3}$, $z_0 = 3 - \frac{8}{3}$

$x_0 = -\frac{2}{3}$, $y_0 = \frac{5}{3}$ $z_0 = \frac{1}{3}$

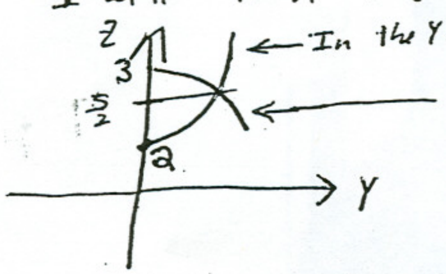
not on the plane.

This is on the plane

$\left(-\frac{2}{3}, \frac{5}{3}, \frac{1}{3}\right)$ verify it is on the plane ✓
the distance from P to it is 4 ✓

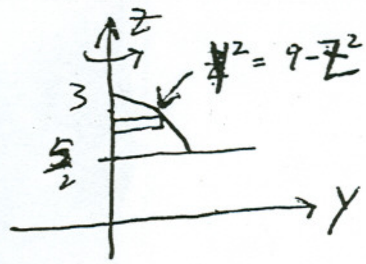
10. The intersection of $x^2 + y^2 + z^2 \leq 9$ and $x^2 + y^2 + (z-5)^2 \leq 9$ is a solid in 3− space. Find the volume of this solid.

These are spherical balls. Each has radius 3. The upper one has center (0 0 5). The lower one has center (0,0,0).
I will treat the intersection as a solid of revolution



← In the yz plane the upper sphere is $y^2 + (z-5)^2 = 9$

In the yz plane the lower sphere is $y^2 + z^2 = 9$



By symmetry, I will revolve and double.

use discs. The vol of a disc is πr^2 thickness
thickness = dz $r = y\text{-coord} = \sqrt{9-z^2}$

$$\text{Vol} = 2\int_{5/2}^{3} \pi(9 - z^2)\,dz$$

$$= 2\pi\left(9z - \frac{z^3}{3}\right)\Big]_{5/2}^{3} = 2\pi\left(27 - 9 - \frac{45}{2} + \frac{125}{24}\right)$$