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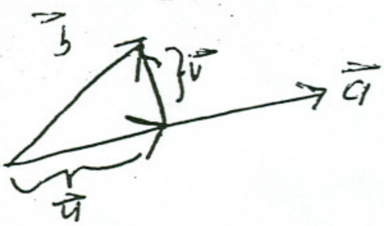
7. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\overrightarrow{a} = 2\overrightarrow{i} + 3\overrightarrow{j} + \overrightarrow{k}$ and $\overrightarrow{b} = 2\overrightarrow{i} - \overrightarrow{j} + 4\overrightarrow{k}$. Find vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ with $\overrightarrow{b} = \overrightarrow{u} + \overrightarrow{v}$, $\overrightarrow{u}$ parallel to $\overrightarrow{a}$, and $\overrightarrow{v}$ perpendicular to $\overrightarrow{a}$.



$\vec u = \text{Proj}_{\vec a}\vec b = \dfrac{\vec a \cdot \vec b}{\vec a \cdot \vec a}\vec a = \dfrac{4-3+4}{4+9+1}\vec a$

$\vec u = \frac{5}{14}(2\vec i + 3\vec j + \vec k)$

$\vec V = \vec b - \vec u = \frac{1}{14}(18\vec i - 29\vec j + 51\vec k)$

verify $\vec u \parallel \vec a$ ✓

$\vec u + \vec v = b$ ✓

$\vec V \cdot \vec a = 0$ ✓

8. Find a point which is the distance 2 from $x + 2y + 2z = 1$.

we want any (x_0, y_0, z_0) so that

$$\frac{|x_0 + 2y_0 + 2z_0 - 1|}{\sqrt{1+4+4}} = 2$$

so we want any (x_0, y_0, z_0) with $x_0 + 2y_0 + 2z_0 - 1 = 6$

any (x_0, y_0, z_0) with $x_0 + 2y_0 + 2z_0 = 7$

so $(7,0,0)$ is an answer

$(1,3,0)$ is an answer

etc.