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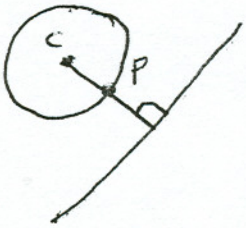
6. Find the equation of the plane which contains the point $(2, 3, 1)$ and is perpendicular to the vector $\overrightarrow{N} = 2\overrightarrow{i} - 3\overrightarrow{j} + 2\overrightarrow{k}$.

$$2(x-2) - 3(y-3) + 2(z-1) = 0$$

7. Find the point on $(x-1)^2 + (y-3)^2 + (z+1)^2 = 14$ which is closest to $x + 2y + 3z = 30$.



Let $C = (1, 3, -1)$.
Let P be the point on the sphere which is closest to the plane.
The vector $\overrightarrow{CP}$ has length $\sqrt{14}$ and points in the direction $+\vec{i} + 2\vec{j} + 3\vec{k}$ or $-(\vec{i} + 2\vec{j} + 3\vec{k})$
·ose vectors have length $\sqrt{14}$
so $\overrightarrow{CP} = \pm(\vec{i} + 2\vec{j} + 3\vec{k})$
so $P = (1+1, 3+2, -1+3)$ or
$P = (1-1, 3-2, -1-3)$

so $P = (2, 5, 2)$ or $(0, 1, -4)$
The distance from $(2, 5, 2)$ to the plane is $\left| \dfrac{2 + 10 + 6 - 30}{\sqrt{14}} \right| = \dfrac{12}{\sqrt{14}}$
The distance from $(0, 1, -4)$ to the plane is $\left| \dfrac{0 + 2 - 12 - 30}{\sqrt{14}} \right| = \dfrac{40}{\sqrt{14}}$

The point on the ~~plane~~ sphere which is closest to the plane is $(2, 5, 2)$