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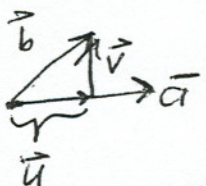
4. Find the angle between $\vec{u} = 3\vec{i} - 2\vec{j} + \vec{k}$ and $\vec{v} = 2\vec{i} + \vec{j} - \vec{k}$.

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|} = \dfrac{6 - 2 - 1}{\sqrt{9+4+1}\,\sqrt{4+1+1}}$

$\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{14}\,\sqrt{6}}\right)$

5. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 4\vec{i} + 6\vec{j} + 4\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$.



$\vec{u} = \text{Proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{\vec{a} \cdot \vec{a}}\vec{a}$

$= \dfrac{4+12+12}{1+4+9}\vec{a} = \dfrac{28}{14}\vec{a}$

$= 2\vec{a} = 2\vec{i} + 4\vec{j} + 6\vec{k} = \vec{u}$

$\vec{v} = \vec{b} - \vec{u} = 2\vec{i} + 2\vec{j} - 2\vec{k} = \vec{v}$

check $\vec{v} \cdot \vec{a} = 2 + 4 - 6 = 0$ ✓