Math 241, Exam 1, Fall, 2020

Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, send a picture of your solutions to

kustin@math.sc.edu

You should KEEP this piece of paper. If possible: put the problems in order before you take your picture. (Use as much paper as necessary).

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

(1) **Find an equation for the plane through the points** $P_1 = (2, 3, 4)$**,** $P_2 = (3, 4, 5)$**, and** $P_3 = (2, -1, 8)$**. Check your answer. Make sure it is correct.**

The vector

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & -4 & 4 \end{vmatrix} = 8\,\vec{i} - 4\,\vec{j} - 4\,\vec{k}$$

is perpendicular to the plane. The plane is

$$8(x-2) - 4(y-3) - 4(z-4) = 0.$$

Divide both sides by 4 to obtain

$$2(x-2) - (y-3) - (z-4) = 0$$

or

$$\boxed{2x - y - z = -3.}$$

Check.

Plug in $P_1 = (2, 3, 4)$ to obtain $2(2) - 3 - 4 = -3\checkmark$.

Plug in $P_2 = (3, 4, 5)$ to obtain $2(3) - 4 - 5 = -3\checkmark$.

Plug in $P_3 = (2, -1, 8)$ to obtain $2(2) - (-1) - 8 = -3\checkmark$.

(2) **Express** $\vec{v} = -\vec{i} + 3\,\vec{j}$ **as the sum of a vector parallel to** $\vec{b} = 3\,\vec{i} - 4\,\vec{j}$ **and a vector perpendicular to** $\vec{b}$**. Check your answer. Make sure it is correct.**

Observe that

$$\operatorname{proj}_{\vec{b}} \vec{v} = \frac{\vec{b} \cdot \vec{v}}{\vec{b} \cdot \vec{b}} \vec{b} = \frac{-3-12}{25}(3\vec{i} - 4\vec{j}) = \frac{-3}{5}(3\vec{i} - 4\vec{j}).$$

It follows that

$$\vec{v} - \operatorname{proj}_{\vec{b}} \vec{v} = (-\vec{i} + 3\vec{j}) + \frac{-3}{5}(3\vec{i} - 4\vec{j}) = \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}.$$

We see that

$\vec{v} = \frac{-3}{5}(3\vec{i} - 4\vec{j}) + \frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$
with $\frac{-3}{5}(3\vec{i} - 4\vec{j})$ parallel to $\vec{b}$ and
$\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$ perpendicular to $\vec{b}$.

Be sure to check that all three assertions are true.

(3) **Let $f(x,y) = 2x^2\sin(2x+3y)$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.**

We calculate

$$\frac{\partial f}{\partial x} = 2x^2 2\cos(2x+3y) + 4x\sin(2x+3y) \quad \text{and} \quad \frac{\partial f}{\partial y} = 6x^2\cos(2x+3y).$$

(4) **Find $\lim\limits_{\substack{(x,y)\to(0,0)\\ \text{along } y=x}} \frac{x^2}{x^2+y^2}$.**

We calculate

$$\lim_{\substack{(x,y)\to(0,0)\\ \text{along } y=x}} \frac{x^2}{x^2+y^2} = \lim_{x\to 0} \frac{x^2}{x^2+x^2} = \lim_{x\to 0} \frac{x^2}{2x^2} = \lim_{x\to 0} \frac{1}{2} = \boxed{\frac{1}{2}}.$$

(5) **Consider the set of all points in 3-space which satisfy both equations $x^2+y^2+z^2=4$ and $x=1$. Describe, name, and draw this set.**

The set of points which satisfy $x^2+y^2+z^2=4$ is the sphere of radius 2 with center at the origin. The set of points which satisfy $x=1$ is the plane which is parallel to the yz-plane, but is one unit in front. The intersection of these two objects is the circle $y^2+z^2=3$ in the plane $x=1$. The center of the circle is $(1,0,0)$; the radius is $\sqrt{3}$. The picture is on the next page.

Picture for number 5