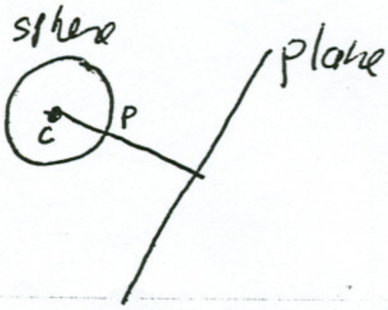
28

6. Find the equation of the plane which contains the point $(1,3,2)$ and is perpendicular to the vector $\overrightarrow{N} = 2\overrightarrow{i} - 3\overrightarrow{j} + 1\overrightarrow{k}$.

$$2(x-1) - 3(y-3) + (z-2) = 0$$

7. Find the point on $(x-4)^2 + (y-7)^2 + (z-8)^2 = 14$ which is closest to $x + 2y + 3z = 0$.



$C = (4,7,8)$

$P = (a,b,c)$

$\overrightarrow{CP} = \langle a-4, b-7, c-8 \rangle$

$\overrightarrow{CP}$ is $\parallel \langle 1,2,3 \rangle$

$\|\overrightarrow{CP}\| = \sqrt{14}$ ← radius of sphere

$\overrightarrow{CP} = \pm \sqrt{14} \dfrac{\langle 1,2,3 \rangle}{\sqrt{14}}$

$\therefore \overrightarrow{CP} = \pm \langle 1,2,3 \rangle$

So either

$a-4 = +1$ or $a-4 = -1$

$b-7 = +2$ $\quad b-7 = -2$

$c-8 = +3$ $\quad c-8 = -3$

So either

$P = (5,9,11)$ or $P = (3,5,5)$

The distance from $(5,9,11)$ to the plane is

$$\frac{|5 + 18 + 33 - 0|}{\sqrt{14}}$$

The distance from $(3,5,5)$ to the plane is

$$\frac{|3 + 10 + 15 - 0|}{\sqrt{14}}$$ ← I am smaller than

The point on the sphere closest to the plane is $(3,5,5)$