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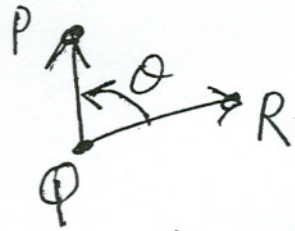
4. Consider the triangle with vertices $P = (1,2,3)$, $Q = (0,1,2)$, and $R = (2,4,7)$. Find the angle of this triangle at the vertex Q.



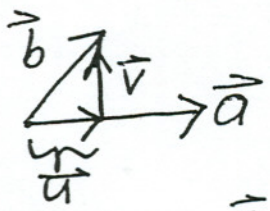
$\vec{QP} \cdot \vec{QR} = \|\vec{QP}\| \, \|\vec{QR}\| \cos\theta$

$\vec{QP} = +\vec{i} + \vec{j} + \vec{k}$

$\vec{QR} = 2\vec{i} + 3\vec{j} + 5\vec{k}$

$\theta = \cos^{-1}\left(\dfrac{\vec{QP} \cdot \vec{QR}}{\|\vec{QP}\| \, \|\vec{QR}\|}\right) = \cos^{-1}\left(\dfrac{2+3+5}{\sqrt{3}\,\sqrt{4+9+25}}\right)$

5. **(There is no partial credit for this problem. Make sure your answer is correct.)** Let $\vec{a} = 2\vec{i} - \vec{j} + 3\vec{k}$ and $\vec{b} = 4\vec{i} - 13\vec{j} + 7\vec{k}$. Find vectors $\vec{u}$ and $\vec{v}$ with $\vec{b} = \vec{u} + \vec{v}$, $\vec{u}$ parallel to $\vec{a}$, and $\vec{v}$ perpendicular to $\vec{a}$. (Every number in the answer is an integer. If you have fractions, either you can rid of them or you have made a mistake.)



$\vec{u} = \text{Proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \dfrac{8+13+21}{4+1+9}\vec{a}$

$= \dfrac{42}{14}\vec{a} = 3\vec{a} = 6\vec{i} - 3\vec{j} + 9\vec{k} = \vec{u}$

$\vec{v} = \vec{b} - \vec{u} = (4\vec{i} - 13\vec{j} + 7\vec{k}) - (6\vec{i} - 3\vec{j} + 9\vec{k})$

$= -2\vec{i} - 10\vec{j} - 2\vec{k} = \vec{v}$

Check

1) $\vec{u} \parallel \vec{a}$ ✓

2) $\vec{u} + \vec{v} = \vec{b}$ ✓

3) $\vec{v} \cdot \vec{a} = -4 + 10 - 6 = 0$