Fall 2000 #1 241

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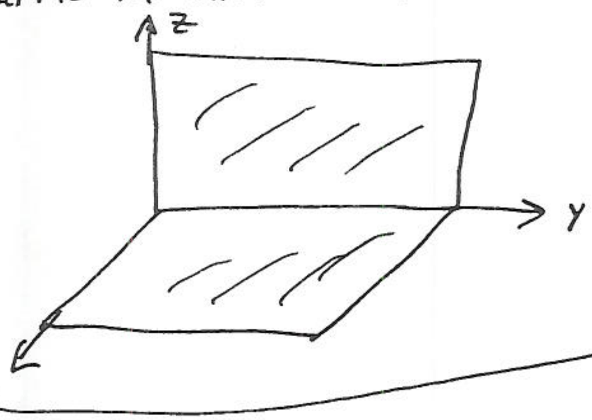
PRINT Your Name:______________________________

There are 10 problems on 4 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

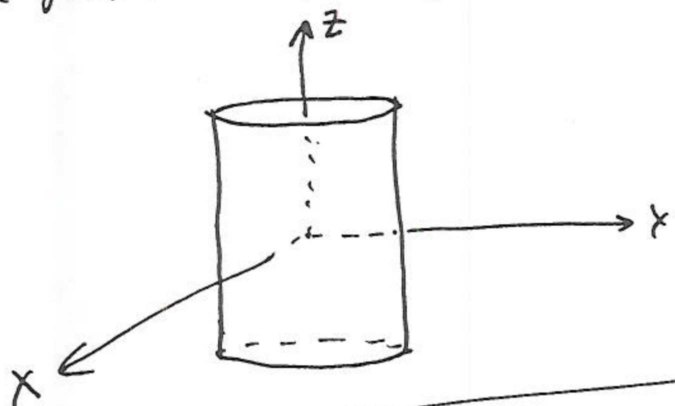
1. Graph and describe the graph of $xz = 0$ in 3− space.

The graph is the union of the yz plane together with the xy plane



2. Graph and describe the graph of $x^2 + y^2 = 4$ in 3− space.

The graph is the cylinder of radius 2 with the z-axis in the center



3. Find the angle between $\vec{u} = 2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{v} = 3\vec{i} + 2\vec{j} + \vec{k}$.

$\vec{u} \cdot \vec{v} = \|\vec{u}\| \, \|\vec{v}\| \cos\theta$

so $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|\,\|\vec{v}\|} = \dfrac{6+6-1}{\sqrt{4+9+1}\sqrt{4+9+1}} = \dfrac{11}{14}$

$\theta = \cos^{-1}\left(\frac{11}{4}\right)$

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4. What is the distance from the point $(1, 2, 3)$ to the z-axis.

The point on the z-axis which is closest to (1 2 3) is (0 0 3). So the distance is $\sqrt{1+4} = \sqrt{5}$

5. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 2x - 6y - 10z + 34 = 0$.

$(x^2+2x+1) + (y^2-6y+9) + (z^2-10z+25) = -34 + 1 + 9 + 25$

$(x+1)^2 + (y-3)^2 + (z-5)^2 = 1$

center $(-1, 3, 5)$
radius 1

6. If $\vec{u} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{v} = 5\vec{i} + y\vec{j} - 3\vec{k}$ are perpendicular, then find y.

$0 = \vec{u} \cdot \vec{v} = 5 + 2y - 9$

$4 = 2y$

$2 = y$

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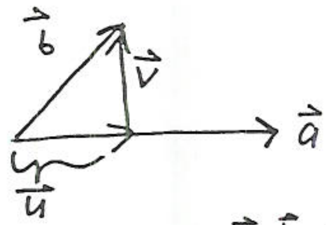
7. (There is no partial credit for this problem. Make sure your answer is correct.) Let $\overrightarrow{a} = \overrightarrow{i} + 3\overrightarrow{j} + 4\overrightarrow{k}$ and $\overrightarrow{b} = 3\overrightarrow{i} + 7\overrightarrow{j} + 7\overrightarrow{k}$. Find vectors $\overrightarrow{u}$ and $\overrightarrow{v}$ with $\overrightarrow{b} = \overrightarrow{u} + \overrightarrow{v}$, $\overrightarrow{u}$ parallel to $\overrightarrow{a}$, and $\overrightarrow{v}$ perpendicular to $\overrightarrow{a}$.



$\vec{u} = proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\vec{a}\cdot\vec{a}}\vec{a} = \frac{3+21+28}{1+9+16}\vec{a} = \frac{52}{26}\vec{a} = 2\vec{a}$

$\vec{u} = 2\vec{i} + 6\vec{j} + 8\vec{k}$

$\vec{V} = \vec{b} - \vec{u} = 3\vec{i} + 7\vec{j} + 7\vec{k} - (2\vec{i} + 6\vec{j} + 8\vec{k})$

$\vec{V} = \vec{i} + \vec{j} - \vec{k}$

check $\vec{a}\cdot\vec{V} = 1+3-4 = 0$ ✓

8. Find the distance between $x + 2y + 3z = 1$ and $x + 2y + 3z = 2$.

$(1, 0, 0)$ is on the first plane

The distance from (100) to the second plane is $\frac{|1+2(0)+3(0)-2|}{\sqrt{1+4+9}} = \frac{1}{\sqrt{14}}$

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9. Find the equation of the plane which contains the point $(1,2,3)$ and is parallel to the plane $2x-3y+4z=2$.

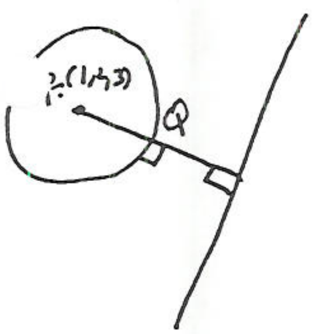
$\vec{N} = 2\vec{i} - 3\vec{j} + 4\vec{k}$ is $\perp$ to both planes

The plane through $(1,2,3)$ $\perp$ to $\vec{N}$ is

$$2(x-1) - 3(y-2) + 4(z-3) = 0$$

$$2x - 3y + 4z = 8$$

10. Find the point on $(x-1)^2+(y-2)^2+(z-3)^2 = 6$ which is closest to $x+y+2z=20$.



Let $P = (1,2,3)$ be the center of the sphere

Let Q be the point on the sphere closest to the plane

so $Q = (a,b,c)$ we must find a,b,c

$\vec{PQ} = (a-1)\vec{i} + (b-2)\vec{j} + (c-3)\vec{k}$

Also $\vec{PQ}$ has length $\sqrt{6}$ and is $\parallel$ to $\vec{i} + \vec{j} + 2\vec{k}$

so $\vec{PQ} = +$ or $-(\vec{i} + \vec{j} + 2\vec{k})$

equate the two formulas for $\vec{PQ}$

either

$a = 1+1$ or $a = 1-1$

$b = 2+1$ $\quad b = 2-1$

$c = 3+2$ $\quad c = 3-2$

The distance from $(2,3,5)$ to the plane is

$$\frac{|2+3+10-20|}{\sqrt{6}} = \frac{5}{\sqrt{6}}$$

The distance from $(0,1,1)$ to the plane is

$$\frac{|0+1+2-20|}{\sqrt{6}} = \frac{17}{\sqrt{6}}$$

So $Q = (2,3,5)$ is the point on the sphere closest to the plane.