**Math 241, Exam 1, Fall, 2017 1:15 class Solutions**

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Tuesday.

**No Calculators, Cell phones, computers, notes, etc.**

(1) **Find a system of parametric equations for the line through the points $P_1 = (1, 2, -1)$ and $P_2 = (-1, 0, 1)$. Check your answer. Make sure it is correct.**

The line is parallel to the vector $\overrightarrow{\boldsymbol{v}} = \overrightarrow{P1P2} = -2\,\overrightarrow{\boldsymbol{i}} - 2\,\overrightarrow{\boldsymbol{j}} + 2\,\overrightarrow{\boldsymbol{k}}$. The line is

$$\boxed{\begin{cases} x = 1 - 2t \\ y = 2 - 2t \\ z = -1 + 2t \end{cases}}$$

Check. At $t = 0$ the line hits $(1, 2, -1)$, which is $P_1$. At $t = 1$ the line hits $(-1, 0, 1)$, which is $P_2$.

(2) **Find an equation for the plane through the points $P_1 = (2, 4, 5)$, $P_2 = (1, 5, 7)$, and $P_3 = (-1, 6, 8)$. Check your answer. Make sure it is correct.**

The plane is perpendicular to the vector

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = (-\overrightarrow{\boldsymbol{i}} + \overrightarrow{\boldsymbol{j}} + 2\overrightarrow{\boldsymbol{k}}) \times (-3\overrightarrow{\boldsymbol{i}} + 2\overrightarrow{\boldsymbol{j}} + 3\overrightarrow{\boldsymbol{k}}) = \begin{vmatrix} \overrightarrow{\boldsymbol{i}} & \overrightarrow{\boldsymbol{j}} & \overrightarrow{\boldsymbol{k}} \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix}$$

$$= (3-4)\overrightarrow{\boldsymbol{i}} - (-3+6)\overrightarrow{\boldsymbol{j}} + (-2+3)\overrightarrow{\boldsymbol{k}} = -\overrightarrow{\boldsymbol{i}} - 3\overrightarrow{\boldsymbol{j}} + \overrightarrow{\boldsymbol{k}}.$$

The plane is

$$-(x-2) - 3(y-4) + (z-5) = 0 \quad or \quad \boxed{-x - 3y + z = -9}.$$

Check.

The plane contains $P_1$ because $-2 - 3(4) + 5 = -9$.

The plane contains $P_2$ because $-1 - 3(5) + 7 = -9$.

The plane contains $P_3$ because $-(-1) - 3(6) + 8 = -9$.

(3) **Express $\overrightarrow{v} = \overrightarrow{i} + 2\overrightarrow{j}$ as the sum of a vector parallel to $\overrightarrow{b} = 3\overrightarrow{i} + 4\overrightarrow{j}$ and a vector orthogonal to $\overrightarrow{b}$. Check your answer. Make sure it is correct.**

Observe that $\overrightarrow{v} = \mathrm{proj}_{\overrightarrow{b}}\overrightarrow{v} + (\overrightarrow{v} - \mathrm{proj}_{\overrightarrow{b}}\overrightarrow{v})$ with $\mathrm{proj}_{\overrightarrow{b}}\overrightarrow{v}$ parallel to $\overrightarrow{b}$ and $(\overrightarrow{v} - \mathrm{proj}_{\overrightarrow{b}}\overrightarrow{v})$ perpendicular to $\overrightarrow{b}$. We compute

$$\mathrm{proj}_{\overrightarrow{b}}\overrightarrow{v} = \frac{\overrightarrow{b}\cdot\overrightarrow{v}}{\overrightarrow{b}\cdot\overrightarrow{b}}\overrightarrow{b} = \frac{3+8}{9+16}(3\overrightarrow{i} + 4\overrightarrow{j}) = \frac{33}{25}\overrightarrow{i} + \frac{44}{25}\overrightarrow{j}$$

So $\overrightarrow{v} = (\frac{33}{25}\overrightarrow{i} + \frac{44}{25}\overrightarrow{j}) + (-\frac{8}{25}\overrightarrow{i} + \frac{6}{25}\overrightarrow{j})$ with $(\frac{33}{25}\overrightarrow{i} + \frac{44}{25}\overrightarrow{j})$ parallel to $\overrightarrow{b}$ and $(-\frac{8}{25}\overrightarrow{i} + \frac{6}{25}\overrightarrow{j})$ perpendicular to $\overrightarrow{b}$.

(4) **Graph and describe the set of points in $3$-space which satisfy $y = 4$ and $x^2 + y^2 + z^2 = 25$.**

The answer is drawn on another page.

(5) **Find the intersection of the line**

$$\begin{cases} x = 2 \\ y = 3 + 2t \\ z = -2 - 2t \end{cases}$$

**and the plane $6x + 3y - 4z = -12$.**

First we find out **when** the line intersects the plane. That is, we solve

$$6(2) + 3(3 + 2t) - 4(-2 - 2t) = -12$$

or $14t = -41$; so $t = \frac{-41}{14}$. Now we find out **where** the intersection occurred: $x = 2$, $y = 3 + 2\left(\frac{-41}{14}\right)$, and $z = -2 - 2\left(\frac{-41}{14}\right)$. The point of intersection is $\boxed{\left(2, -\frac{20}{7}, \frac{27}{7}\right)}$.

<u>Check.</u> The point $(2, -\frac{20}{7}, \frac{27}{7})$ is on the plane because

$$6(2) + 3\left(-\frac{20}{7}\right) - 4\left(\frac{27}{7}\right) = 12 - \frac{60 + 108}{7} = 12 - 24 = -12.\checkmark$$