Math 241, Exam 1, Fall, 2017 11:40 class Solutions

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. Please make your work coherent, complete, and correct. Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

The exams will be returned on Tuesday.

No Calculators, Cell phones, computers, notes, etc.

(1) **Find a system of parametric equations for the line through the points** $P_1 = (1, 2, 0)$ **and** $P_2 = (1, 1, -1)$**. Check your answer. Make sure it is correct.**

The vector $\vec{v} = \overrightarrow{P_1P_2} = -\vec{j} - \vec{k}$ is parallel to our line; the point $P_1 = (1, 2, 0)$ is on our line. Our line is

$$\boxed{\begin{cases} x = 1 \\ y = 2 - t \\ z = -t \end{cases}}$$

Check. When $t = 0$ our equations give the point $(1, 2, 0)$ which is P_1. At $t = 1$ our equations give the point $(1, 1, -1)$ which is P_2.

(2) **Find an equation for the plane through the points** $P_1 = (1, -1, 2)$**,** $P_2 = (2, 1, 3)$**, and** $P_3 = (-1, 2, -1)$**. Check your answer. Make sure it is correct.**

The plane is perpendicular to

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ -2 & 3 & -3 \end{vmatrix} = (-6-3)\vec{i} - (-3+2)\vec{j} + (3+4)\vec{k}.$$

Our line is

$$-9(x-1) + (y+1) + 7(z-2) = 0 \quad \text{or} \quad \boxed{-9x + y + 7z = 4.}$$

Check. See if the points work the proposed answer:

$$\begin{gathered} -9(1) - 1 + 7(2) = 4\checkmark \\ -9(2) + 1 + 7(3) = 4\checkmark \\ -9(-1) + 2 + 7(-1) = 4\checkmark \end{gathered}$$

(3) **Express $\overrightarrow{v} = 4\overrightarrow{i} + \overrightarrow{j}$ as the sum of a vector parallel to $\overrightarrow{b} = 2\overrightarrow{i} + 3\overrightarrow{j}$ and a vector orthogonal to $\overrightarrow{b}$. Check your answer. Make sure it is correct.**

Observe that

$$\text{proj}_{\overrightarrow{b}}\,\overrightarrow{v} = \frac{\overrightarrow{b}\cdot\overrightarrow{v}}{\overrightarrow{b}\cdot\overrightarrow{b}}\overrightarrow{b} = \frac{8+3}{4+9}(2\overrightarrow{i} + 3\overrightarrow{j}).$$

Hence,

$$\boxed{\begin{array}{c}\overrightarrow{v} = (\tfrac{22}{13}\overrightarrow{i} + \tfrac{33}{13}\overrightarrow{j}) + (\tfrac{30}{13}\overrightarrow{i} - \tfrac{20}{13}\overrightarrow{j}) \\ \text{with } (\tfrac{22}{13}\overrightarrow{i} + \tfrac{33}{13}\overrightarrow{j}) \text{ parallel to } \overrightarrow{b} \text{ and } (\tfrac{30}{13}\overrightarrow{i} - \tfrac{20}{13}\overrightarrow{j}) \text{ perpendicular to } \overrightarrow{b}\end{array}}$$

(4) **Find the center and radius of the sphere**

$$x^2 + y^2 + z^2 - 8x - 6y + 10z + 34 = 0.$$

Complete the square. The sphere is

$$x^2 - 8x + \boxed{16} + y^2 - 6y + \boxed{\boxed{9}} + z^2 + 10z + \boxed{\boxed{\boxed{25}}} = -34 + \boxed{16} + \boxed{\boxed{9}} + \boxed{\boxed{\boxed{25}}}.$$

$$(x-4)^2 + (y-3)^2 + (z+5)^2 = 16$$

$\boxed{\text{The sphere has center } (4,3,-5) \text{ and radius } 4.}$

(5) **Find the distance from the point $(2,1,3)$ and the line**

$$\begin{cases} x = 5 + 2t \\ y = 1 + 6t \\ z = 3. \end{cases}$$

The point $P_0 = (5,1,3)$ is on the line. The vector $\overrightarrow{v} = 2\overrightarrow{i} + 6\overrightarrow{j}$ is parallel to the line. Please look at the picture which appears in a separate file. The distance from $P_1 = (2,1,3)$ to the line is

$$|\overrightarrow{P_0P_1} - \text{proj}_{\overrightarrow{v}}\,\overrightarrow{P_0P_1}| = |-3\overrightarrow{i} - \frac{\overrightarrow{P_0P_1}\cdot\overrightarrow{v}}{\overrightarrow{v}\cdot\overrightarrow{v}}\overrightarrow{v}| = |-3\overrightarrow{i} - \frac{-6}{40}(2\overrightarrow{i} + 6\overrightarrow{j})|$$

$$= |-3\overrightarrow{i} + \frac{3}{10}(\overrightarrow{i} + 3\overrightarrow{j})| = |\frac{-27}{10}\overrightarrow{i} + \frac{9}{10}\overrightarrow{j}| = \frac{9}{10}|-3\overrightarrow{i} + \overrightarrow{j}|$$

$$= \boxed{\frac{9}{10}\sqrt{10}}.$$