**Math 241, Exam 1, Spring, 2022**

**You should KEEP this piece of paper.** Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

**No Calculators, Cell phones, computers, notes, etc.**

(1) **Find a system of parametric equations for the line through the points** $P_1 = (1, 2, 3)$ **and** $P_2 = (-1, 4, 9)$**. Check your answer. Make sure it is correct.**

The vector $\overrightarrow{P_1P_2} = -2\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 6\vec{\boldsymbol{k}}$. The line through $P_1 = (1, 2, 3)$ and parallel to $\overrightarrow{P_1P_2} = -2\vec{\boldsymbol{i}} + 2\vec{\boldsymbol{j}} + 6\vec{\boldsymbol{k}}$ is

$$\boxed{\begin{cases} x = 1 - 2t \\ y = 2 + 2t \\ z = 3 + 6t \end{cases}}$$

**Check:** When $t = 0$, our proposed answer gives $(1, 2, 3)$.✓When $t = 1$, our proposed answer gives $(-1, 4, 9)$.✓

(2) **Find an equation for the plane through the points** $P_1 = (1, 2, 3)$**,** $P_2 = (6, 5, 4)$**, and** $P_3 = (-1, 0, 1)$**. Check your answer. Make sure it is correct.**

We calculate $\overrightarrow{P_1P_2} = 5\vec{\boldsymbol{i}} + 3\vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$, $\overrightarrow{P_1P_3} = -2\vec{\boldsymbol{i}} - 2\vec{\boldsymbol{j}} - 2\vec{\boldsymbol{k}}$, and

$$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix} \vec{\boldsymbol{i}} & \vec{\boldsymbol{j}} & \vec{\boldsymbol{k}} \\ 5 & 3 & 1 \\ -2 & -2 & -2 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 1 \\ -2 & -2 \end{vmatrix} \vec{\boldsymbol{i}} - \begin{vmatrix} 5 & 1 \\ -2 & -2 \end{vmatrix} \vec{\boldsymbol{j}} + \begin{vmatrix} 5 & 3 \\ -2 & -2 \end{vmatrix} \vec{\boldsymbol{k}}$$

$$= -4\vec{\boldsymbol{i}} + 8\vec{\boldsymbol{j}} - 4\vec{\boldsymbol{k}}.$$

The plane through $(1, 2, 3)$ perpendicular to $-4\vec{i} + 8\vec{j} - 4\vec{k}$ is

$$-4(x-1) + 8(y-2) - 4(z-3) = 0.$$

Divide by $-4$: $(x-1) - 2(y-2) + (z-3) = 0$. Expand to get

$$x - 1 - 2y + 4 + z - 3 = 0$$

or $\boxed{x - 2y + z = 0}$.

**Check:** The point $P_1$ satisfies our proposed answer because $1-4+3=0$. The point $(6, 5, 4)$ satisfies our proposed answer because $6 - 10 + 4 = 0$. The point $(-1, 0, 1)$ satisfies our proposed answer because $-1 - 1 = 0$.

(3) **Express $\vec{v} = 3\vec{i} + 5\vec{j}$ as the sum of a vector parallel to $\vec{b} = 2\vec{i} + 3\vec{j}$ and a vector orthogonal to $\vec{b}$. Check your answer. Make sure it is correct.**

We compute

$$\text{proj}_{\vec{b}}\,\vec{v} = \frac{\vec{v}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b} = \frac{6+15}{4+9}(2\vec{i} + 3\vec{j}) = \frac{42}{13}\vec{i} + \frac{63}{13}\vec{j}.$$

We also compute

$$\vec{v} - \text{proj}_{\vec{b}}\,\vec{v} = -\frac{3}{13}\vec{i} + \frac{2}{13}\vec{j}.$$

We see that

$$\boxed{\begin{array}{l}\vec{v} = (\frac{42}{13}\vec{i} + \frac{63}{13}\vec{j}) + (-\frac{3}{13}\vec{i} + \frac{2}{13}\vec{j}),\\ \text{with } (\frac{42}{13}\vec{i} + \frac{63}{13}\vec{j}) \text{ parallel to } \vec{b}\\ \text{and } (-\frac{3}{13}\vec{i} + \frac{2}{13}\vec{j}) \text{ perpendicular to } \vec{b}.\end{array}}$$

(4) **Put $2x^2 + 3y^2 + 4z^2 - 12x - 24y - 40z + 158 = 0$ in the form**

$$A(x-x_0)^2 + B(y-y_0)^2 + C(z-z_0)^2 = D,$$

**where $x_0$, $y_0$, $z_0$, $A$, $B$, $C$, and $D$ are numbers.**

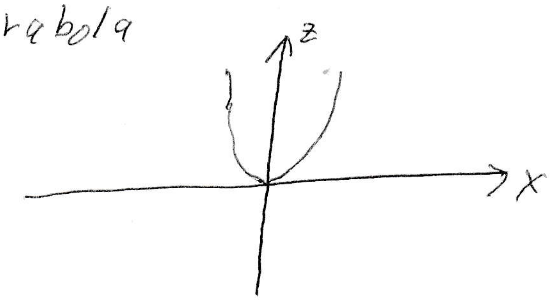
The original equation has the same solution set as

$$2(x^2 - 6x + 9) + 3(y^2 - 8y + 16) + 4(z^2 - 10z + 25) = -158 + 18 + 48 + 100$$

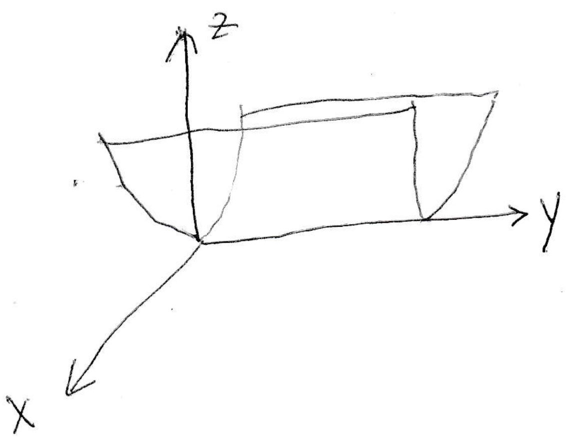
$$\boxed{2(x-3)^2 + 3(y-4)^2 + 4(z-5)^2 = 8.}$$

(5) **Name, describe, and graph the set of all points in three-space which satisfy the equation $z = x^2$.**

5. In the xz-plane the set of points which satisfy $z = x^2$ is a parabola




In 3-space, one gets the same picture for each $y$, so the set of points which satisfy $z = x^2$ is a parabolic cylinder




This is the surface one would obtain by taking a large piece of sheet metal and folding one edge into a parabola.