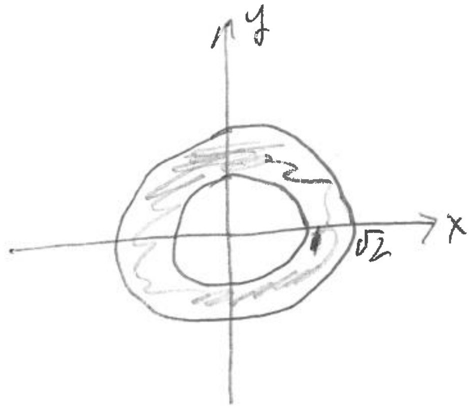
15.7, number 77: Find the volume of the solid cut from the thick-walled cylinder $1 \leq x^2 + y^2 \leq 2$ by the cones $z = \pm\sqrt{x^2 + y^2}$.

Answer: We drew the fattest cross section on the next page. This shows us that the volume is equal to

$$
\begin{aligned}
&\int_0^{2\pi} \int_1^{\sqrt{2}} \int_{-r}^{r} r \, dz \, dr \, d\theta \\
&= \int_0^{2\pi} \int_1^{\sqrt{2}} rz \Big|_{-r}^{r} \, dr \, d\theta \\
&= \int_0^{2\pi} \int_1^{\sqrt{2}} 2r^2 \, dr \, d\theta \\
&= \int_0^{2\pi} \frac{2}{3} r^3 \Big|_1^{\sqrt{2}} \, d\theta \\
&= \int_0^{2\pi} \frac{2}{3} (2\sqrt{2} - 1) \, d\theta \\
&= \boxed{2\pi \frac{2}{3} (2\sqrt{2} - 1)}
\end{aligned}
$$

Picture 15.7 Number 77

For each fixed point (r, θ) in the shaded region



z goes from $z = -r$ to $z = +r$

Of course, the shaded region is $0 \leq \theta \leq 2\pi$ $\quad 1 \leq r \leq \sqrt{2}$

All together the integral is

$$\int_0^{2\pi} \int_1^{\sqrt{2}} \int_{-r}^{r} r \, dz \, dr \, d\theta$$