15.7, number 39: Set up the iterated integral for evaluating

$$\iiint_D f(r,\theta,z) r \, dz \, dr \, d\theta$$

over the solid region D, where D is the right circular cylinder whose base is the region inside the cardiod $r = 1 + \cos\theta$ and outside the circle $r = 1$ and whose top lies in the plane $z = 4$.

Answer: All cross sections of D parallel to the xy-plane look exactly the same; so the integral looks like

$$\iint_{\text{cross section}} \int_0^4 f(r,\theta,z) r \, dz \, dr \, d\theta.$$

We drew the cross-section parallel to the xy-plane on the next page and we used this picture to see that

$$\iint_{\text{cross section}} r \, dr \, d\theta = \int_{-\pi/2}^{\pi/2} \int_1^{1+\cos\theta} r \, dr \, d\theta.$$

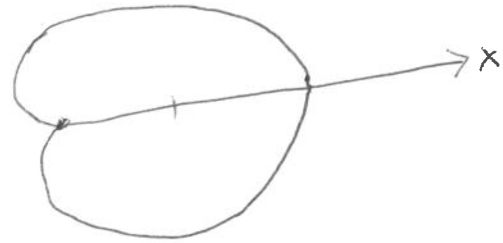
We conclude that the answer is

$$\boxed{\int_{-\pi/2}^{\pi/2} \int_1^{1+\cos\theta} \int_0^4 f(r,\theta,z) r \, dz \, dr \, d\theta.}$$

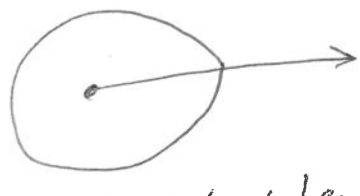
Picture 15.7 Number 39

We draw the cardiod $r = 1 + \cos\theta$

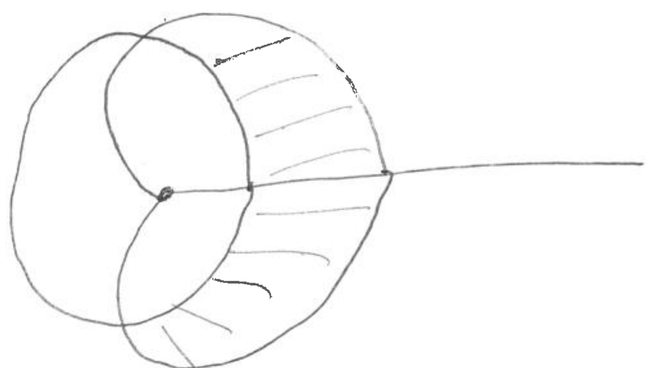
θ	r
0	2
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	1
2π	2



We draw $r = 1$



The two graphs intersect when $1 = 1 + \cos\theta$ so $0 = \cos\theta$ $\theta = \frac{\pi}{2}$ and $-\frac{\pi}{2}$



We shaded the region inside the cardiod and outside the circle

For each fixed angle with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$,
r does from $r = 1$ to $r = 1 + \cos\theta$

To integrate over this region one uses

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{1}^{1+\cos\theta} r\,dr\,d\theta$$