15.7, number 37: Set up the iterated integral for evaluating

$$\iiint_D f(r,\theta,z)r\,dz\,dr\,d\theta$$

over the solid region D, where D is the right circular cylinder whose base is the circle $r = 2\sin\theta$ in the xy-plane and whose top lies in the plane $z = 4 - y$.

Answer:

Keep in mind that the top, which is $z = 4 - y$, becomes $z = 4 - 4\sin\theta$ in polar coordinates.

All cross sections of D parallel to the xy-plane look exactly the same; so the integral looks like

$$\iint_{\text{cross section}} \int_0^{4-r\sin\theta} f(r,\theta,z)r\,dz\,dr\,d\theta.$$

We drew the cross-section parallel to the xy-plane on the next page. Maybe you want to multiply both sides of $r = 2\sin\theta$ by r to see that $r^2 = 2r\sin\theta$ or $x^2 + y^2 = 2y$ or $x^2 + (y-1)^2 = 1$. In other words, $r = 2\sin\theta$ is the circle of radius 1 with center $(0,1)$. Maybe you want to plot some points to reach the same conclusion. Do notice though that you travel around the whole circle once as θ goes from 0 to π. I plotted the points on the next page. At any rate, to integrate over the cross section in polar coordinates one uses

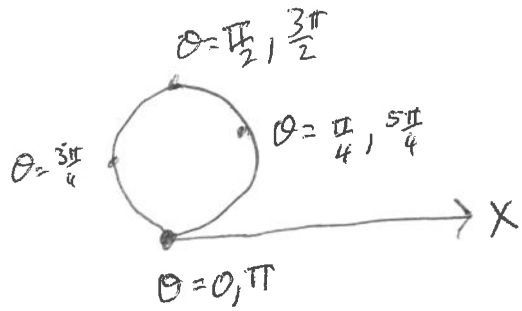
$$\int_0^{\pi}\int_0^{2\sin\theta} \;---\; r\,dr\,d\theta.$$

Put both thoughts together to see that the answer is

$$\boxed{\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{4-r\sin\theta} f(r,\theta,z)r\,dz\,dr\,d\theta.}$$

Picture 15.7 Number 37

Plot $r = 2\sin\theta$



For each fixed θ with $0 \le \theta \le \pi$,

r goes from $r = 0$ to $r = 2\sin\theta$

To integrate over this circle one uses

$$\int_0^{\pi} \int_0^{2\sin\theta} \underline{\qquad} \; r\,dr\,d\theta$$