15.7, number 23: Compute

$$\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta.$$

Answer:

$$\begin{aligned}
&\textstyle\int_0^{2\pi} \int_0^1 \int_r^{\sqrt{2-r^2}} r \, dz \, dr \, d\theta \\
&= \textstyle\int_0^{2\pi} \int_0^1 rz \Big|_r^{\sqrt{2-r^2}} dr \, d\theta \\
&= \textstyle\int_0^{2\pi} \int_0^1 r(\sqrt{2-r^2} - r) dr \, d\theta \\
&= \textstyle\int_0^{2\pi} \left(\frac{1}{-2} \frac{2}{3} (2-r^2)^{3/2} - \frac{r^3}{3} \right) \Big|_0^1 d\theta \\
&= \textstyle\int_0^{2\pi} \left(\frac{1}{-2} \frac{2}{3} (2-1)^{3/2} - \frac{1}{3} - \frac{1}{-2} \frac{2}{3} (2)^{3/2} \right) d\theta \\
&= \textstyle\int_0^{2\pi} \frac{2}{3} (\sqrt{2} - 1) \, d\theta = \boxed{\frac{4\pi}{3} (\sqrt{2} - 1)}
\end{aligned}$$