

15.5, number 9: Compute

$$\int_1^e \int_1^{e^2} \int_1^{e^3} \frac{1}{xyz} dx dy dz.$$

Answer:

$$\begin{aligned} & \int_1^e \int_1^{e^2} \int_1^{e^3} \frac{1}{xyz} dx dy dz \\ &= \int_1^e \int_1^{e^2} \frac{1}{yz} \ln x \Big|_1^{e^3} dy dz \\ &= \int_1^e \int_1^{e^2} \frac{1}{yz} (\ln(e^3) - \ln(1)) dy dz \\ &= \int_1^e \int_1^{e^2} \frac{1}{yz} 3 dy dz \\ &= \int_1^e \frac{1}{z} \ln y \Big|_1^{e^2} 3 dz \\ &= 6 \int_1^e \frac{1}{z} dz \\ &= 6 \ln z \Big|_1^e \\ &= \boxed{6} \end{aligned}$$