15.5, number 41: Compute

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dx\, dy\, dz.$$

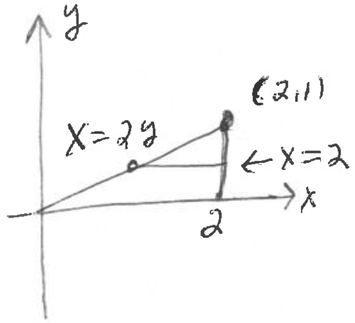
Answer: None of us know how to integrate $\cos(x^2)$. So we must do something clever. I propose that we exchange the order of integrating with respect to x and y. The pictures on the next page show that

$$\begin{aligned}
& \textstyle\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\, dx\, dy\, dz \\
= & \textstyle\int_0^4 \int_0^2 \int_0^{x/2} \frac{4\cos(x^2)}{2\sqrt{z}}\, dy\, dx\, dz \\
= & \textstyle\int_0^4 \int_0^2 \frac{4\cos(x^2)}{2\sqrt{z}} y\Big|_0^{x/2}\, dx\, dz \\
= & \textstyle\int_0^4 \int_0^2 \frac{2x\cos(x^2)}{2\sqrt{z}}\, dx\, dz \\
= & \textstyle\int_0^4 \frac{\sin(x^2)}{2\sqrt{z}}\Big|_0^2\, dz \\
= & \textstyle\int_0^4 \frac{\sin(4)}{2\sqrt{z}}\, dz \\
= & \sin(4)\sqrt{z}\Big|_0^4 \\
= & \boxed{2\sin(4)}
\end{aligned}$$

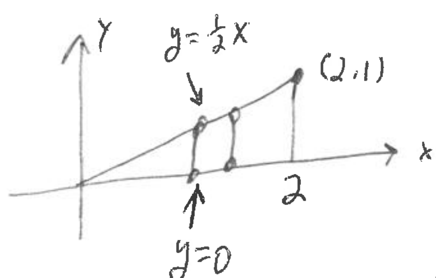
Picture 15.5 Number 41

The integral $\int_0^1 \int_{2y}^{2} dx\, dy$ is taken over

For each fixed y with $0 \le y \le 1$, x goes from $x = 2y$ to $x = 1$



If we switch the order of integration we have



The integral is $\int_0^2 \int_0^{\frac{1}{2}x} dy\, dx$