15.5, number 35: Find the volume of the region cut from the solid elliptical cylinder $x^2 + 4y^2 \leq 4$ by the xy-plane and the plane $z = x + 2$.

Answer: We find the volume of a solid with base in the xy-plane and top given by $z = x + 2$. We integrate over the base of the top. The base is everything inside the ellipse $x^2 + 4y^2 = 4$. See the next page.

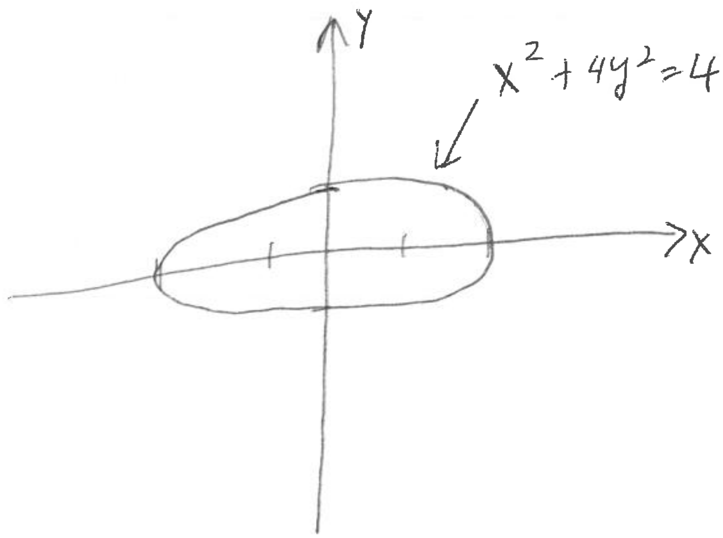
To integrate over the ellipse, we have to do Trig substitution. That is unpleasant (and unnecessary), so let us change varibles so thatwe are integrating over a circle. The full story on changing variables involves taking four partial deviatives (see 15.8); but we are only changing one variable. We let $v = 2y$; so $dv = 2dy$.

The volume is equal to

$$
\begin{aligned}
& \iint_{\text{inside } x^2 + 4y^2 = 4} (x+2)\, dx\, dy \\
= & \ \tfrac{1}{2} \iint_{\text{inside } x^2 + v^2 = 4} (x+2)\, dx\, dv \\
= & \ \tfrac{1}{2} \int_0^{2\pi} \int_0^2 (r\cos\theta + 2) r\, dr\, d\theta \\
= & \ \tfrac{1}{2} \int_0^{2\pi} \int_0^2 (r^2\cos\theta + 2r)\, dr\, d\theta \\
= & \ \tfrac{1}{2} \int_0^{2\pi} \left(\tfrac{r^3}{3}\cos\theta + r^2\right)\Big|_0^2\, d\theta \\
= & \ \tfrac{1}{2} \int_0^{2\pi} \left(\tfrac{8}{3}\cos\theta + 4\right) d\theta \\
= & \ \tfrac{1}{2} \left(\tfrac{8}{3}\sin\theta + 4\theta\right)\Big|_0^{2\pi} \\
= & \ \boxed{4\pi}
\end{aligned}
$$

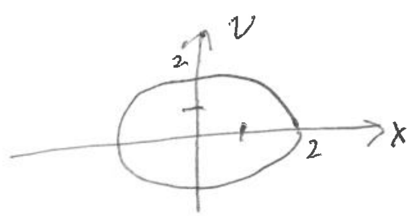
Picture 15.5 Number 35

The base is



Let $v = 2y$

In x, v coordinates the base is $x^2+v^2=4$



If we do polar coordinates here

$$\left(\begin{array}{l} x = r\cos\theta \\ v = r\sin\theta \end{array} \right),$$

then $0 \le \theta \le 2\pi$ and $0 \le r \le 2$