

15.5, number 23: Find the volume of the region between $z = y^2$ and the xy -plane that is bounded by the planes $x = 0$, $x = 1$, $y = -1$, and $y = 1$.

Answer: The parabolic cylinder $z = y^2$ is the top. The xy -plane is the bottom. The planes $x = 0$, $x = 1$, $y = -1$, and $y = 1$ are the sides. We can find the volume as a double integral over the base (which is the rectangle bounded by the lines $x = 0$, $x = 1$, $y = -1$, and $y = 1$ in the xy -plane) of the height. I'll fill up the base with vertical lines. For each fixed x between 0 and 1; y goes from -1 to 1. The volume is

$$\begin{aligned} & \int_0^1 \int_{-1}^1 y^2 dy dx \\ &= \int_0^1 \left. \frac{y^3}{3} \right|_{-1}^1 dx \\ &= \int_0^1 \frac{2}{3} dx = \boxed{\frac{2}{3}} \end{aligned}$$